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Exam P Study Manual

3rd Edition

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Contents

Lesson 1

Sets

Most things in life are not certain. Probability is a mathematical model for uncertain events. Probability assigns a number between 0 and 1 to each event. This number may have the following meanings:

1. It may indicate that of all the events in the universe, the proportion of them included in this event is that number. For example, if one says that 70% of the population owns a car, it means that the number of people owning a car is 70% of the number of people in the population.

2. It may indicate that in the long run, this event will occur that proportion of the time. For example, if we say that a certain medicine cures an illness 80% of the time, it means that we expect that if we have a large number of people, let's say 1000, with that illness who take the medicine, approximately 800 will be cured.

From a mathematical viewpoint, probability is a function from the space of events to the interval of real numbers between 0 and 1. We write this function as $P[A]$, where A is an event. We often want to study combinations of events. For example, if we are studying people, events may be "male", "female", "married", and "single". But we may also want to consider the event "young and married", or "male or single". To understand how to manipulate combinations of events, let's briefly study set theory. An event can be treated as a set.

A set is a collection of objects. The objects in the set are called members of the set. Two special sets are

1. The entire space. I'll use Ω for the entire space, but there is no standard notation. All members of all sets must come from Ω.

2. The empty set, usually denoted by $\varnothing$. This set has no members.

There are three important operations on sets:

Union If A and B are sets, we write the union as $A \cup B$. It is defined as the set whose members are all the members of A plus all the members of B. Thus if x is in $A \cup B$, then either x is in A or x is in B. x may be a member of both A and B. The union of two sets is always at least as large as each of the two component sets.

Intersection If A and B are sets, we write the intersection as $A \cap B$. It is defined as the set whose members are in both A and B. The intersection of two sets is always no larger than each of the two component sets.

Complement If A is a set, its complement is the set of members of Ω that are not members of A. There is no standard notation for complement; different textbooks use A', A^c, and $\bar{A}$. I'll use A', the notation used in SOA sample questions. Interestingly, SOA sample solutions use A^c instead.

Venn diagrams are used to portray sets and their relationships. Venn diagrams display a set as a closed figure, usually a circle or an ellipse, and different sets are shown as intersecting if they have common elements. We present three Venn diagrams here, each showing a function of two sets as a shaded region. Figure 1.1 shows the union of two sets, A and B. Figure 1.2 shows the intersection of A and B. Figure 1.3 shows the complement of $A \cup B$. In these diagrams, A and B have a non-trivial intersection. However, if A and B are two sets with no intersection, we say that A and B are *mutually exclusive*. In symbols, mutually exclusive means $A \cap B = \varnothing$.

Important set properties are:

1. Associative property: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$

2. Distributive property: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. Distributive property for complement: $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

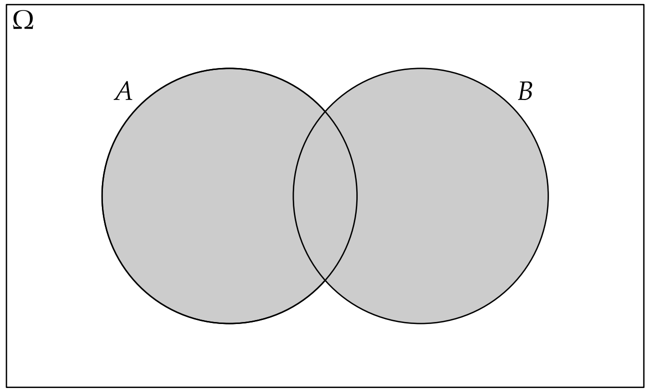


Figure 1.1: $A \cup B$

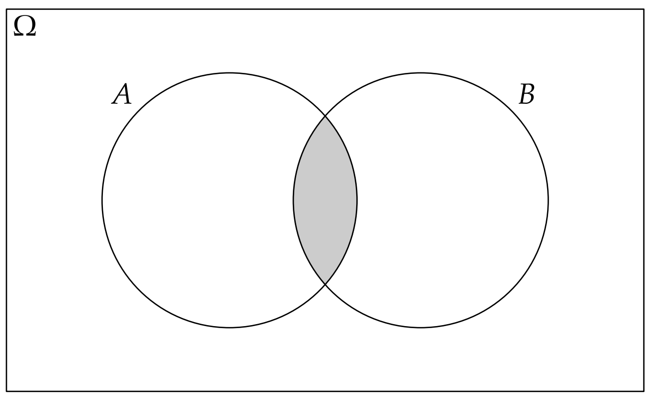


Figure 1.2: $A \cap B$

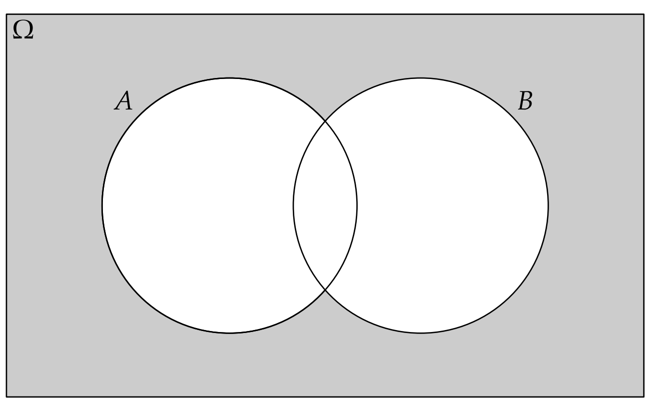


Figure 1.3: $(A \cup B)'$

Example 1A Simplify $(A \cup B) \cap (A \cup B')$.

Solution: By the distributive property,

$$(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$$

But B and B' are mutually exclusive: $B \cap B' = \varnothing$. So

$$(A \cup B) \cap (A \cup B') = A \cup \varnothing = \boxed{A}$$

□

Probability theory has three axioms:

1. The probability of any set is greater than or equal to 0.
2. The probability of the entire space is 1.
3. The probability of a countable union of mutually exclusive sets is the sum of the probabilities of the sets.

From these axioms, many properties follow, such as:

1. $P[A] \le 1$ for any A.
2. $P[A'] = 1 - P[A]$.
3. $P[A \cap B] \le P[A]$.

Looking at Figure 1.1, we see that $A \cup B$ has three mutually exclusive components: $A \cap B'$, $B \cap A'$, and the intersection of the two sets $A \cap B$. To compute $P[A \cup B]$, if we add together $P[A]$ and $P[B]$, we double count the intersection, so we must subtract its probability. Thus

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \tag{1.1}$$

This can also be expressed with $\cup$ and $\cap$ reversed:

$$P[A \cap B] = P[A] + P[B] - P[A \cup B] \tag{1.2}$$

Example 1B A company is trying to plan social activities for its employees. It finds:

(i) 35% of employees do not attend the company picnic.
(ii) 80% of employees do not attend the golf and tennis outing.
(iii) 25% of employees do not attend the company picnic and also don't attend the golf and tennis outing.

What percentage of employees attend both the company picnic and the golf and tennis outing?

Solution: Let A be the event of attending the picnic and B the event of attending the golf and tennis outing. Then

$$\begin{aligned}
P[A \cap B] &= P[A] + P[B] - P[A \cup B] \\
P[A] &= 1 - P[A'] = 1 - 0.35 = 0.65 \\
P[B] &= 1 - P[B'] = 1 - 0.80 = 0.20 \\
P[A \cup B] &= 1 - P[(A \cup B)'] = 1 - P[A' \cap B'] = 1 - 0.25 = 0.75 \\
P[A \cap B] &= 0.65 + 0.20 - 0.75 = \boxed{\mathbf{0.10}}
\end{aligned}$$

□

Equations (1.1) and (1.2) are special cases of *inclusion-exclusion equations*. The generalization deals with unions or intersections of any number of sets. For the probability of the union of n sets, add up the probabilities of the sets, then subtract the probabilities of intersections of 2 sets, add the probabilities of intersections of 3 sets, and so on, until you get to n:

$$\begin{aligned}
P\left[\bigcup_{i=1}^{n} A_i\right] = \sum_{i=1}^{n} P[A_i] - \sum_{i \ne j} P\left[A_i \cap A_j\right] + \sum_{i \ne j \ne k} P\left[A_i \cap A_j \cap A_k\right] \\
- \cdots + (-1)^{n-1} P[A_1 \cap A_2 \cap \cdots \cap A_n]
\end{aligned} \tag{1.3}$$

Table 1.1: Formula summary for probabilities of sets

Set properties

$$(A \cup B) \cup C = A \cup (B \cup C) \text{ and } (A \cap B) \cap C = A \cap (B \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \text{ and } A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$(A \cup B)' = A' \cap B' \text{ and } (A \cap B)' = A' \cup B'$$

Mutually exclusive: $A \cap B = \varnothing$
$P[A \cup B] = P[A] + P[B]$ for A and B mutually exclusive

Inclusion-exclusion equations:

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \tag{1.1}$$
$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$$

On an exam, it is unlikely you would need this formula for more than 3 sets. With 3 sets, there are probabilities of three intersections of two sets to subtract and one intersection of all three sets to add:

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$$

Example 1C Your company is trying to sell additional policies to group policyholders. It finds:

(i) 10% of customers do not have group life, group health, or group disability.
(ii) 25% of customers have group life.
(iii) 75% of customers have group health.
(iv) 20% of customers have group disability.
(v) 40% of customers have group life and group health.
(vi) 22% of customers have group disability and group health.
(vii) 5% of customers have group life and group disability.

Calculate the percentage of customers who have all three coverages: group life, group health, and group disability.

Solution: Each insurance coverage is an event, and we are given intersections of events, so we'll use inclusion-exclusion on the union. The first statement implies that the probability of the union of all three events is $1 - 0.1 = 0.9$. Let the probability of the intersection, which is what we are asked for, be x. Then

$$0.9 = 0.25 + 0.75 + 0.20 - 0.40 - 0.22 - 0.05 + x = 0.53 + x$$

It follows that $x = \boxed{0.37}$. □

Most exam questions based on this lesson will require use of the inclusion-exclusion equations for 2 or 3 sets.

Exercises

1.1. **[110-S83:17]** If $P[X] = 0.25$ and $P[Y] = 0.80$, then which of the following inequalities must be true?

I. $P[X \cap Y] \le 0.25$

II. $P[X \cap Y] \ge 0.20$

III. $P[X \cap Y] \ge 0.05$

(A) I only (B) I and II only (C) I and III only (D) II and III only
(E) The correct answer is not given by (A) , (B) , (C) , or (D) .

1.2. **[110-S85:29]** Let E and F be events such that $P[E] = \frac{1}{2}$, $P[F] = \frac{1}{2}$, and $P[E' \cap F'] = \frac{1}{3}$. Then $P[E \cup F'] =$

(A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$ (E) 1

1.3. **[110-S88:10]** If E and F are events for which $P[E \cup F] = 1$, then $P[E' \cup F']$ must equal

(A) 0
(B) $P[E'] + P[F'] - P[E']P[F']$
(C) $P[E'] + P[F']$
(D) $P[E'] + P[F'] - 1$
(E) 1

1.4. **[110-W96:23]** Let A and B be events such that $P[A] = 0.7$ and $P[B] = 0.9$.
Calculate the largest possible value of $P[A \cup B] - P[A \cap B]$.

(A) 0.20 (B) 0.34 (C) 0.40 (D) 0.60 (E) 1.60

1.5. You are given that $P[A \cup B] - P[A \cap B] = 0.3$, $P[A] = 0.8$, and $P[B] = 0.7$.
Determine $P[A \cup B]$.

1.6. **[S01:12, Sample:3]** You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$.
Calculate $P[A]$.

(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.8

1.7. **[1999 Sample:1]** A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house.

Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

(A) 0.4 (B) 0.5 (C) 0.6 (D) 0.7 (E) 0.9

Exercises continue on the next page . . .

1.8. [**S03:1**,**Sample:1**] A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A) 24 (B) 36 (C) 41 (D) 52 (E) 60

1.9. A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched baseball but neither soccer nor gymnastics during the last year.

1.10. An insurance company finds that among its policyholders:

(i) Each one has either health, dental, or life insurance.
(ii) 81% have health insurance.
(iii) 36% have dental insurance.
(iv) 24% have life insurance.
(v) 5% have all three insurance coverages.
(vi) 14% have dental and life insurance.
(vii) 12% have health and life insurance.

Determine the percentage of policyholders having health insurance but not dental insurance.

1.11. [**Sample:243**] An insurance agent's files reveal the following facts about his policyholders:

i) 243 own auto insurance.
ii) 207 own homeowner insurance.
iii) 55 own life insurance and homeowner insurance.
iv) 96 own auto insurance and homeowner insurance.
v) 32 own life insurance, auto insurance and homeowner insurance.
vi) 76 more clients own only auto insurance than only life insurance.
vii) 270 own only one of these three insurance products.

Calculate the total number of the agent's policyholders who own at least one of these three insurance products.

(A) 389 (B) 407 (C) 423 (D) 448 (E) 483

Exercises continue on the next page . . .

1.12. [**Sample:244**] A profile of the investments owned by an agent's clients follows:

i) 228 own annuities.
ii) 220 own mutual funds.
iii) 98 own life insurance and mutual funds.
iv) 93 own annuities and mutual funds.
v) 16 own annuities, mutual funds, and life insurance.
vi) 45 more clients own only life insurance than own only annuities.
vii) 290 own only one type of investment (i.e., annuity, mutual fund, or life insurance).

Calculate the agent's total number of clients.

(A) 455 (B) 495 (C) 496 (D) 500 (E) 516

1.13. [**S00:1**,**Sample:2**] The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35% . Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work.

Calculate the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A) 0.05 (B) 0.12 (C) 0.18 (D) 0.25 (E) 0.35

1.14. In a certain town, there are 1000 cars. All cars are white, blue, or gray, and are either sedans or SUVs. There are 300 white cars, 400 blue cars, 760 sedans, 180 white sedans, and 320 blue sedans.

Determine the number of gray SUVs.

1.15. [**F00:3**,**Sample:5**] An auto insurance company has 10,000 policyholders. Each policyholder is classified as

(i) young or old;
(ii) male or female; and
(iii) married or single

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

Calculate the number of the company's policyholders who are young, female, and single.

(A) 280 (B) 423 (C) 486 (D) 880 (E) 896

1.16. An auto insurance company has 10,000 policyholders. Each policyholder is classified as

(i) young or old;
(ii) male or female; and
(iii) married or single

Of these policyholders, 4000 are young, 5600 are male, and 3500 are married. The policyholders can also be classified as 2820 young males, 1540 married males, and 1300 young married persons. Finally, 670 of the policyholders are young married males.

How many of the company's policyholders are old, female, and single?

1.17. [**F01:9**,**Sample:8**] Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist.

Calculate the probability that a randomly chosen member of this group visits a physical therapist.

(A) 0.26 (B) 0.38 (C) 0.40 (D) 0.48 (E) 0.62

Exercises continue on the next page . . .

1.18. For new hires in an actuarial student program:

(i) 20% have a postgraduate degree.
(ii) 30% are Associates.
(iii) 60% have 2 or more years of experience.
(iv) 14% have both a postgraduate degree and are Associates.
(v) The proportion who are Associates and have 2 or more years of experience is twice the proportion who have a postgraduate degree and have 2 or more years experience.
(vi) 25% do not have a postgraduate degree, are not Associates, and have less than 2 years of experience.
(vii) Of those who are Associates and have 2 or more years experience, 10% have a postgraduate degree.

Calculate the percentage that have a postgraduate degree, are Associates, and have 2 or more years experience.

1.19. [**Sample:246**] An actuary compiles the following information from a portfolio of 1000 homeowners insurance policies:

(i) 130 policies insure three-bedroom homes.
(ii) 280 policies insure one-story homes.
(iii) 150 policies insure two-bath homes.
(iv) 30 policies insure three-bedroom, two-bath homes.
(v) 50 policies insure one-story, two-bath homes.
(vi) 40 policies insure three-bedroom, one-story homes.
(vii) 10 policies insure three-bedroom, one-story, two-bath homes.

Calculate the number of homeowners policies in the portfolio that insure neither one-story nor two-bath nor three-bedroom homes.

(A) 310 (B) 450 (C) 530 (D) 550 (E) 570

1.20. [**S03:5,Sample:9**] An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.
(ii) 70% of the customers insure more than one car.
(iii) 20% of the customers insure a sports car.
(iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

(A) 0.13 (B) 0.21 (C) 0.24 (D) 0.25 (E) 0.30

1.21. An employer offers employees the following coverages:

(i) Vision insurance
(ii) Dental insurance
(iii) Long term care (LTC) insurance

Employees who enroll for insurance must enroll for at least two coverages. You are given

(i) The probability of enrolling for vision insurance is 40%.
(ii) The probability of enrolling for dental insurance is 80%.
(iii) The probability of enrolling for LTC insurance is 70%.
(iv) The probability of enrolling for all three insurances is 20%.

Calculate the probability of not enrolling for any insurance.

Exercises continue on the next page . . .

1.22. [**Sample:126**] Under an insurance policy, a maximum of five claims may be filed per year by a policyholder. Let $p(n)$ be the probability that a policyholder files n claims during a given year, where $n = 0, 1, 2, 3, 4, 5$. An actuary makes the following observations:

(i) $p(n) \geq p(n + 1)$ for 0, 1, 2, 3, 4.
(ii) The difference between $p(n)$ and $p(n + 1)$ is the same for $n = 0, 1, 2, 3, 4$.
(iii) Exactly 40% of policyholders file fewer than two claims during a given year.

Calculate the probability that a random policyholder will file more than three claims during a given year.

(A) 0.14 (B) 0.16 (C) 0.27 (D) 0.29 (E) 0.33

1.23. [**Sample:128**] An insurance agent offers his clients auto insurance, homeowners insurance and renters insurance. The purchase of homeowners insurance and the purchase of renters insurance are mutually exclusive. The profile of the agent's clients is as follows:

(i) 17% of the clients have none of these three products.
(ii) 64% of the clients have auto insurance.
(iii) Twice as many of the clients have homeowners insurance as have renters insurance.
(iv) 35% of the clients have two of these three products.
(v) 11% of the clients have homeowners insurance, but not auto insurance.

Calculate the percentage of the agent's clients that have both auto and renters insurance.

(A) 7% (B) 10% (C) 16% (D) 25% (E) 28%

1.24. [**Sample:134**] A mattress store sells only king, queen and twin-size mattresses. Sales records at the store indicate that one-fourth as many queen-size mattresses are sold as king and twin-size mattresses combined. Records also indicate that three times as many king-size mattresses are sold as twin-size mattresses.

Calculate the probability that the next mattress sold is either king or queen-size.

(A) 0.12 (B) 0.15 (C) 0.80 (D) 0.85 (E) 0.95

1.25. [**Sample:143**] The probability that a member of a certain class of homeowners with liability and property coverage will file a liability claim is 0.04, and the probability that a member of this class will file a property claim is 0.10. The probability that a member of this class will file a liability claim but not a property claim is 0.01.

Calculate the probability that a randomly selected member of this class of homeowners will not file a claim of either type.

(A) 0.850 (B) 0.860 (C) 0.864 (D) 0.870 (E) 0.890

1.26. [**Sample:146**] A survey of 100 TV watchers revealed that over the last year:

(i) 34 watched CBS.
(ii) 15 watched NBC.
(iii) 10 watched ABC.
(iv) 7 watched CBS and NBC.
(v) 6 watched CBS and ABC.
(vi) 5 watched NBC and ABC.
(vii) 4 watched CBS, NBC, and ABC.
(viii) 18 watched HGTV and of these, none watched CBS, NBC, or ABC.

Calculate how many of the 100 TV watchers did not watch any of the four channels (CBS, NBC, ABC or HGTV).

(A) 1 (B) 37 (C) 45 (D) 55 (E) 82

Exercises continue on the next page . . .

1.27. **[Sample:179]** This year, a medical insurance policyholder has probability 0.70 of having no emergency room visits, 0.85 of having no hospital stays, and 0.61 of having neither emergency room visits nor hospital stays.

Calculate the probability that the policyholder has at least one emergency room visit and at least one hospital stay this year.

(A) 0.045 (B) 0.060 (C) 0.390 (D) 0.667 (E) 0.840

1.28. **[Sample:198]** In a certain group of cancer patients, each patient's cancer is classified in exactly one of the following five stages: stage 0, stage 1, stage 2, stage 3, or stage 4.

i) 75% of the patients in the group have stage 2 or lower.
ii) 80% of the patients in the group have stage 1 or higher.
iii) 80% of the patients in the group have stage 0, 1, 3, or 4.

One patient from the group is randomly selected.

Calculate the probability that the selected patient's cancer is stage 1.

(A) 0.20 (B) 0.25 (C) 0.35 (D) 0.48 (E) 0.65

1.29. **[Sample:207]** A policyholder purchases automobile insurance for two years. Define the following events:

F = the policyholder has exactly one accident in year one.
G = the policyholder has one or more accidents in year two.

Define the following events:

i) The policyholder has exactly one accident in year one and has more than one accident in year two.
ii) The policyholder has at least two accidents during the two-year period.
iii) The policyholder has exactly one accident in year one and has at least one accident in year two.
iv) The policyholder has exactly one accident in year one and has a total of two or more accidents in the two-year period.
v) The policyholder has exactly one accident in year one and has more accidents in year two than in year one.

Determine the number of events from the above list of five that are the same as $F \cap G$.

(A) None
(B) Exactly one
(C) Exactly two
(D) Exactly three
(E) All

1.30. **[F01:1,Sample:4]** An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44.

Calculate the number of blue balls in the second urn.

(A) 4 (B) 20 (C) 24 (D) 44 (E) 64

Exercises continue on the next page . . .

1.31. [**Sample:254**] The annual numbers of thefts a homeowners insurance policyholder experiences are analyzed over three years. Define the following events:

(i) A = the event that the policyholder experiences no thefts in the three years.
(ii) B = the event that the policyholder experiences at least one theft in the second year.
(iii) C = the event that the policyholder experiences exactly one theft in the first year.
(iv) D = the event that the policyholder experiences no thefts in the third year.
(v) E = the event that the policyholder experiences no thefts in the second year, and at least one theft in the third year.

Determine which three events satisfy the condition that the probability of their union equals the sum of their probabilities.

(A) Events A, B, and E
(B) Events A, C, and E
(C) Events A, D, and E
(D) Events B, C, and D
(E) Events B, C, and E

1.32. [**S01:31**,**Sample:15**] An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$, respectively.

Determine the probability that a randomly chosen employee will choose no supplementary coverage.

(A) 0 (B) $\frac{47}{144}$ (C) $\frac{1}{2}$ (D) $\frac{97}{144}$ (E) $\frac{7}{9}$

1.33. [**Sample:255**] Four letters to different insureds are prepared along with accompanying envelopes. The letters are put into the envelopes randomly.

Calculate the probability that at least one letter ends up in its accompanying envelope.

(A) 27/256 (B) 1/4 (C) 11/24 (D) 5/8 (E) 3/4

1.34. The probability of event U is 0.8 and the probability of event V is 0.4.

What is the lowest possible probability of the event $U \cap V$?

Solutions

1.1. Since $X \cap Y \subset X$, $P[X \cap Y] \le P[X] = 0.25$. And since $P[X \cup Y] \le 1$ and $P[X \cup Y] = P[X] + P[Y] - P[X \cap Y]$, it follows that

$$\begin{aligned} P[X] + P[Y] - P[X \cap Y] &\le 1 \\ 0.25 + 0.80 - P[X \cap Y] &\le 1 \end{aligned}$$

so $P[X \cap Y] \ge 0.05$. One can build a counterexample to II by arranging for the union of X and Y to equal the entire space. **(C)**

1.2. Split $E \cup F'$ into the following two disjoint sets: E and $E' \cap F'$. These two sets are disjoint since $E \cap E' = \varnothing$, and comprise $E \cup F'$ because everything in E is included and anything in F' is either in E or is in $E' \cap F'$.

$$P[E \cup F'] = P[E] + P[E' \cap F'] = \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6}} \qquad \textbf{(D)}$$

1.3.

$$\begin{aligned} P[E' \cup F'] &= P[E'] + P[F'] - P[E' \cap F'] \\ E' \cap F' &= (E \cup F)' \\ P[E' \cap F'] &= 1 - P[E \cup F] = 0 \\ P[E' \cup F'] &= P[E'] + P[F'] \qquad \textbf{(C)} \end{aligned}$$

1.4. $P[A \cup B] = P[A] + P[B] - P[A \cap B]$. From this equation, we see that for fixed A and B, the smaller $P[A \cap B]$ is, the larger $P[A \cup B]$ is. Therefore, maximizing $P[A \cup B]$ also maximizes $P[A \cup B] - P[A \cap B]$. The highest possible value of $P[A \cup B]$ is 1. Then

$$P[A \cap B] = P[A] + P[B] - P[A \cup B] = 0.7 + 0.9 - 1 = 0.6$$

and

$$P[A \cup B] - P[A \cap B] = 1 - 0.6 = \boxed{\mathbf{0.4}} \qquad \textbf{(C)}$$

1.5. $P[A \cap B] = P[A] + P[B] - P[A \cup B]$, so $P[A \cap B] = 0.8 + 0.7 - P[A \cup B] = 1.5 - P[A \cup B]$. Then substituting into the first probability that we are given, $2P[A \cup B] - 1.5 = 0.3$, so $P[A \cup B] = \boxed{\mathbf{0.9}}$.

1.6. The union of $A \cup B$ and $A \cup B'$ is the entire space, since $B \cup B' = \Omega$, the entire space. The probability of the union is 1. By the inclusion-exclusion principle

$$P[(A \cup B) \cap (A \cup B')] = 0.7 + 0.9 - 1 = 0.6$$

Now,

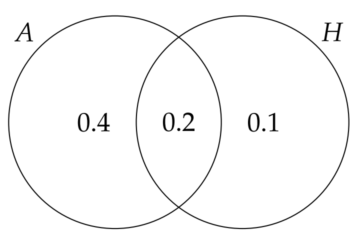
$$(A \cup B) \cap (A \cup B') = A \cap (B \cup B') = A$$

so $P[A] = \boxed{\mathbf{0.6}}$ **(D)**

1.7. If we let A be the set of automobile owners and H the set of house owners, we want

$$\big(P[A] - P[A \cap H]\big) + \big(P[H] - P[A \cap H]\big) = P[A] + P[H] - 2P[A \cap H]$$

Using what we are given, this equals $0.6 + 0.3 - 2(0.2) = \boxed{\mathbf{0.5}}$. **(B)**



1.8. If the sets of watching gymnastics, baseball, and soccer are G, B and S respectively, we want $G' \cap B' \cap S' = (G \cup B \cup S)'$, and

$$\begin{aligned} P[G \cup B \cup S] &= P[G] + P[B] + P[S] - P[G \cap B] - P[G \cap S] - P[B \cap S] + P[G \cap B \cap S] \\ &= 0.28 + 0.29 + 0.19 - 0.14 - 0.12 - 0.10 + 0.08 = 0.48 \end{aligned}$$

So the answer is $1 - 0.48 = \boxed{\mathbf{0.52}}$. **(D)**

1.9. The percentage watching baseball is 29%. Of these, 14% also watched gymnastics and 12% also watched soccer, so we subtract these. However, in this subtraction we have double counted those who watch all three sports (8%), so we add that back in. The answer is $29\% - 14\% - 12\% + 8\% = \boxed{\mathbf{11\%}}$.

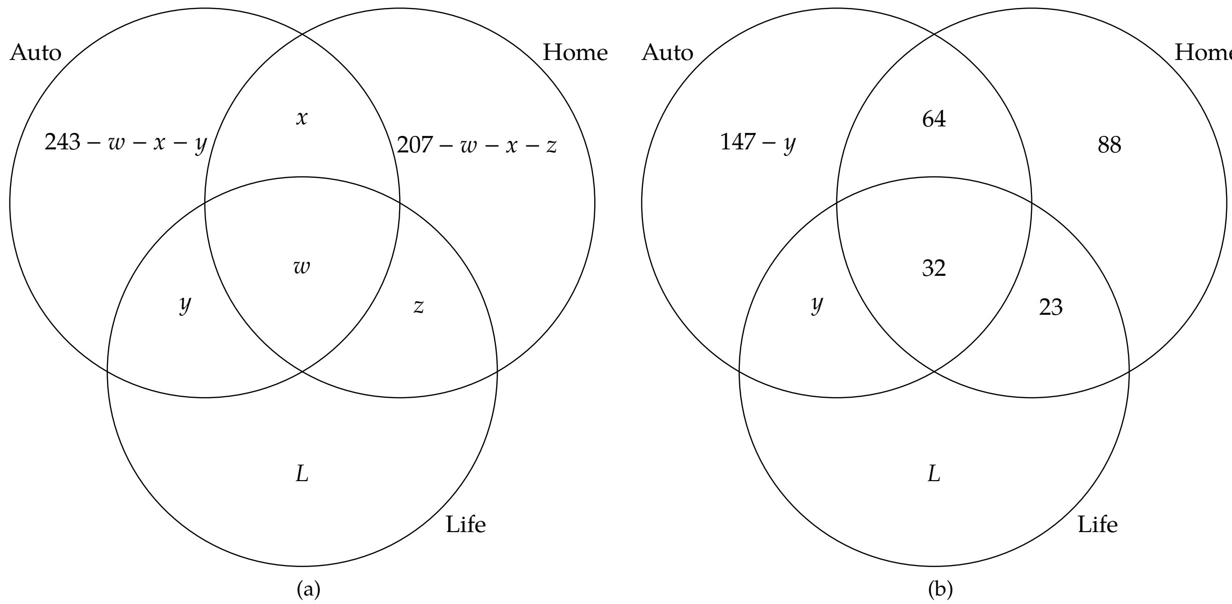


Figure 1.4: Venn diagrams for exercise 1.11

1.10. Since everyone has insurance, the union of the three insurances has probability 1. If we let H, D, and L be health, dental, and life insurance respectively, then

$$\begin{aligned}1 &= P[H \cup D \cup L]\\ &= P[H] + P[D] + P[L] - P[H \cap D] - P[H \cap L] - P[D \cap L] + P[H \cap D \cap L]\\ &= 0.81 + 0.36 + 0.24 - P[H \cap D] - 0.14 - 0.12 + 0.05\\ &= 1.20 - P[H \cap D]\end{aligned}$$

so $P[H \cap D] = 0.20$. Since 81% have health insurance, this implies that $0.81 - 0.20 = \boxed{\mathbf{61\%}}$ have health insurance but not dental insurance.

1.11. A Venn diagram is helpful here. Figure 1.4a incorporates i) and ii). Based on iii), iv), and v), $w + z = 55$, $w + x = 96$, and $w = 32$, resulting in Figure 1.4b. Based on vi), $147 - y = L + 76$, so $L = 71 - y$, as in Figure 1.5a. Now we use vii) to solve for y and L:

$$\begin{aligned}147 - y + 88 + 71 - y &= 270\\ 306 - 2y &= 270\\ y &= 18\end{aligned}$$

Now we have Figure 1.5b. We add up all the numbers to get

$$129 + 64 + 88 + 18 + 32 + 23 + 53 = \boxed{\mathbf{407}} \qquad \textbf{(B)}$$

1.12. Virtually identical to the previous exercise, and you can use the same Venn diagrams with updated numbers. We have $98 - 16 = 82$ with only life insurance and mutual funds and $93 - 16 = 77$ with only annuities and mutual funds. The number with mutual funds only is $220 - 77 - 16 - 82 = 45$. Let y be the number with annuities and life insurance only, and L the number with life insurance only. The number with annuities only is $228 - 93 - y = 135 - y$. Then from vi),

$$135 - y = L - y - 45$$

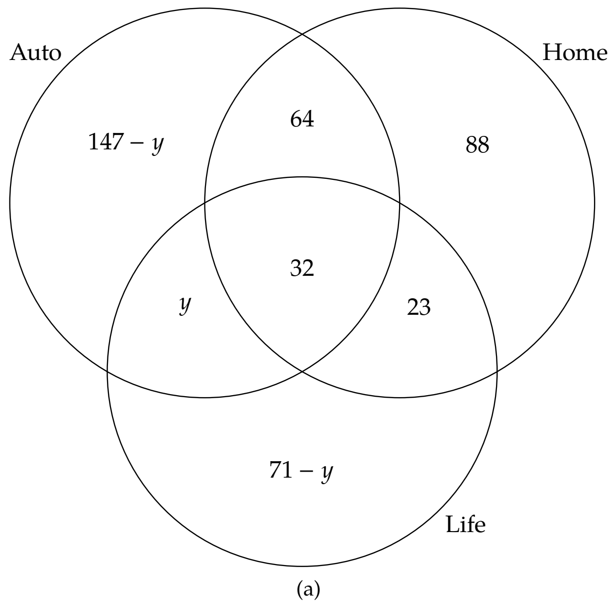


Figure 1.5: Venn diagrams for exercise 1.11

$$L = 180 - y$$

The number From vii), $135 - y + 45 + 180 - y = 290$ so $y = 35$. The number with annuities only is $135 - y = 100$. The number with life insurance only is $180 - y = 145$. Adding up all the numbers,

$$100 + 45 + 145 + 77 + 35 + 82 + 16 = \boxed{500} \qquad \textbf{(D)}$$

1.13. If lab work is A and specialist is B, then $P[A' \cap B'] = 0.35$, $P[A] = 0.3$, and $P[B] = 0.4$. We want $P[A \cap B]$. Now, $P(A \cup B)' = P(A' \cap B') = 0.35$, so $P[A \cup B] = 0.65$. Then

$$P[A \cap B] = P[A] + P[B] - P[A \cup B] = 0.3 + 0.4 - 0.65 = \boxed{0.05} \qquad \textbf{(A)}$$

1.14. There are $1000 - 760 = 240$ SUVs. Of these, there are $300 - 180 = 120$ white SUVs and $400 - 320 = 80$ blue SUVs, so there must be $240 - 120 - 80 = \boxed{40}$ gray SUVs.

The following table may be helpful. Given numbers are in roman and derived numbers are in italics.

	Total	Sedan	SUV
Total	1000	760	*240*
White	300	180	*120*
Blue	400	320	*80*
Gray			*40*

1.15. There are 3000 young. Of those, remove 1320 young males and 1400 young marrieds. That removes young married males twice, so add back young married males. The result is $3000 - 1320 - 1400 + 600 = \boxed{880}$. **(D)**

1.16. If the classifications are A, B and C for young, male, and married respectively, we calculate (# denotes the number of members of a set.)

$$\#[A' \cap B' \cap C'] = \#[(A \cup B \cup C)']$$
$$\#[A \cup B \cup C] = \#[A] + \#[B] + \#[C] - \#[A \cap B] - \#[A \cap C] - \#[B \cap C] + \#[A \cap B \cap C]$$

$$= 4000 + 5600 + 3500 - 2820 - 1540 - 1300 + 670 = 8110$$
$$\#[A' \cap B' \cap C'] = 10{,}000 - 8{,}110 = \boxed{\mathbf{1{,}890}}$$

1.17. Let A be physical therapist and B be chiropractor. We want $P[A]$. We are given that $P[A \cap B] = 0.22$ and $P[A' \cap B'] = 0.12$. Also, $P[B] = P[A] + 0.14$. Then $P[A \cup B] = 1 - P[A' \cap B'] = 0.88$. So

$$\begin{aligned} P[A \cap B] &= P[A] + P[B] - P[A \cup B] \\ 0.22 &= P[A] + P[A] + 0.14 - 0.88 \\ P[A] &= \boxed{\mathbf{0.48}} \qquad \textbf{(D)} \end{aligned}$$

1.18. Let A be postgraduate degree, B Associates, and C 2 or more years experience. Let $x = P[B \cap C]$. Then

$$\begin{aligned} P[A' \cap B' \cap C'] &= 0.25 \\ P[(A \cup B \cup C)'] &= 0.25 \\ P[A \cup B \cup C] &= 0.75 \end{aligned}$$
$$\begin{aligned} 0.75 &= P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C] \\ &= 0.2 + 0.3 + 0.6 - 0.14 - 0.5x - x + 0.1x \\ &= 0.96 - 1.4x \\ x &= 0.15 \end{aligned}$$

The answer is $0.1(0.15) = 0.015$, or $\boxed{\mathbf{1.5\%}}$.

1.19. This is a straightforward inclusion-exclusion principle question: to get total in at least one of the three classes, add those in 1, subtract those in 2, add those in 3:

$$130 + 280 + 150 - 30 - 50 - 40 + 10 = 450$$

That leaves $\boxed{\mathbf{550}}$ in none of the classes. **(D)**

1.20. Statement (iv) in conjunction with statement (ii) tells us that $0.15(0.7) = 0.105$ insured more than one car including a sports car. Then from (iii), $0.2 - 0.105 = 0.095$ insure one car that is a sports car. Since 0.3 insure one car, that leaves $0.3 - 0.095 = \boxed{\mathbf{0.205}}$ who insure one car that is not a sports car. **(B)**

1.21. If we let A, B, C be vision, dental, and LTC, then

$$P[A' \cap B' \cap C'] = P[(A \cup B \cup C)'] = 1 - P[A \cup B \cup C]$$

Since nobody enrolls for just one coverage,

$$P[A] = P[A \cap B] + P[A \cap C] - P[A \cap B \cap C]$$

and similar equations can be written for $P[B]$ and $P[C]$. Adding the three equations up,

$$\begin{aligned} P[A \cap B] + P[A \cap C] + P[B \cap C] &= \frac{P[A] + P[B] + P[C] + 3P[A \cap B \cap C]}{2} \\ &= \frac{0.4 + 0.8 + 0.7 + 3(0.2)}{2} = 1.25 \end{aligned}$$

Using the inclusion-exclusion formula,

$$\begin{aligned} P[A \cup B \cup C] &= P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C] \\ &= 0.4 + 0.8 + 0.7 - 1.25 + 0.2 = 0.85 \end{aligned}$$

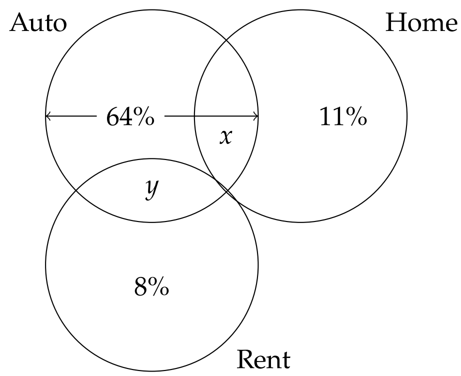
and the answer is $1 - 0.85 = \boxed{\mathbf{0.15}}$.

1.22. Set $d = p(0) + p(1) - p(2) - p(3)$. We are given that $p(0) + p(1) = 0.4$. So $p(2) + p(3) = 0.4 - d$. By (ii), $p(2)+p(3)-p(4)-p(5) = d$. So $p(4)+p(5) = p(2)+p(3)-d = 0.4-2d$. The six probabilities $p(0), p(1), \ldots, p(5)$ must add up to 1. Therefore

$$\begin{aligned} 0.4 + (0.4 - d) + 0.4 - 2d) &= 1 \\ 1.2 - 3d &= 1 \\ d &= \frac{1}{15} \end{aligned}$$

The probability of 4 or 5 claims is $0.4 - 2/15 = \boxed{\mathbf{0.266667}}$. **(C)**

1.23. A Venn diagram may be helpful here.



Let x be the proportion with both auto and home and y the proportion with both auto and renters. We are given that 17% have no product, so 83% have at least one product. 64% have auto and 11% have homeowners but not auto, so that leaves 83% − 64% − 11% = 8% who have only renters insurance. Now we can set up two equations in x and y. From (iv), $x + y = 0.35$. From (iii), $0.11 + x = 2(0.08 + y)$. Solving,

$$\begin{aligned} 0.11 + (0.35 - y) &= 0.16 + 2y \\ 0.46 - 0.16 &= 3y \\ y &= \boxed{\mathbf{0.10}} \qquad \textbf{(B)} \end{aligned}$$

1.24. We'll use K, Q, and T for the three events king-size, queen-size, and twin-size mattresses. We're given $P[K] + P[T] = 4P[Q]$ and $P[K] = 3P[T]$, and the three events are mutually exclusive and exhaustive so their probabilities must add up to 1, $P[K] + P[T] + P[Q] = 1$. We have three equations in three unknowns. Let's solve for $P[T]$ and then use the complement.

$$\begin{aligned} 3P[T] + P[T] = 4P[Q] &= 4(1 - 4P[T]) \\ 4P[T] &= 4 - 16P[T] \\ P[T] &= 0.2 \end{aligned}$$

and the answer is $1 - 0.2 = \boxed{\mathbf{0.8}}$. **(C)**

1.25. We want to calculate the probability of filing some claim, or the union of those filing property and liability claims, because then we can calculate the probability of the complement of this set, those who file no claim. To calculate the probability of filing some claim, we need the probability of filing both types of claim.

The probability that a member will file a liability claim is 0.04, and of these 0.01 do not file a property claim so 0.03 do file both claims. The probability of the union of those who file liability and property claims is the sum of the probabilities of filing either type of claim, minus the probability of filing both types of claim, or $0.04+0.10-0.03 = 0.11$, so the probability of not filing either type of claim is $1 - 0.11 = \boxed{\mathbf{0.89}}$. **(E)**

1.26. First let's calculate how many did not watch CBS, NBC, or ABC. As usual, we add up the ones who watched one, minus the ones who watched two, plus the ones who watched three:

$$34 + 15 + 10 - 7 - 6 - 5 + 4 = 45$$

An additional 18 watched HGTV for a total of 63 who watched something. $100 - 63 = \boxed{37}$ watched nothing. **(B)**

1.27. Let A be the event of an emergency room visit and B the event of a hospital stay. We have $P[A] = 1 - 0.7 = 0.3$ and $P[B] = 1 - 0.85 = 0.15$. Also $P[A \cup B] = 1 - 0.61 = 0.39$. Then the probability we want is

$$P[A \cap B] = P[A] + P[B] - P[A \cup B] = 0.3 + 0.15 - 0.39 = \boxed{0.06} \qquad \textbf{(B)}$$

1.28. From ii), the probability of stage 0 is 20%. From iii), the probability of stage 2 is 20%. From i), the probability of stage 0, 1, or 2 is 75%. So the probability of stage 1 is $0.75 - 2(0.2) = \boxed{0.35}$. **(C)**

1.29. $F \cap G$ is the event of exactly one accident in year one and at least one in year two.

i) This event is not the same since it excludes the event of one in year one and one in year two. ✗

ii) This event is not the same since it includes two in year one and none in year two, among others. ✗

iii) This event is the same. ✓

iv) This event is the same, since to have two or more accidents total, there must be one or more accidents in year two. ✓

v) This event is not the same since it excludes having one in year one and one in year two. ✗

(C)

1.30. Let x be the number of blue balls in the second urn. Then the probability that both balls are red is $0.4(16)/(16+x)$ and the probability that both balls are blue is $0.6x/(16+x)$. The sum of these two expressions is 0.44, so

$$\begin{aligned}
\frac{6.4 + 0.6x}{16 + x} &= 0.44 \\
6.4 + 0.6x &= 7.04 + 0.44x \\
0.16x &= 0.64 \\
x &= \boxed{4} \qquad \textbf{(A)}
\end{aligned}$$

1.31. We want to find three mutually exclusive events. We search for two mutually exclusive events, then look for a third one exclusive to the other two. Event A is a good start, since any event with a theft is exclusive to it, and only event D does not have a theft. Looking at B, C, and E, we see that B has one theft in the second year and E doesn't, so they're mutually exclusive, and both are exclusive to A. **(A)**

1.32. $P[A] = P[A \cap B] + P[A \cap C]$, since the only way to have coverage A is in combination with exactly one of B and C. Similarly $P[B] = P[B \cap A] + P[B \cap C]$ and $P[C] = P[C \cap A] + P[C \cap B]$. Adding up the three equalities, we get

$$P[A] + P[B] + P[C] = 2\big(P[A \cap B] + P[A \cap C] + P[B \cap C]\big) = \frac{1}{4} + \frac{1}{3} + \frac{5}{12} = 1$$

so the sum of the probabilities of two coverages is 0.5. But the only alternative to two coverages is no coverage, so the probability of no coverage is $1 - 0.5 = \boxed{0.5}$. **(C)**

1.33. Use the inclusion-exclusion principle with 4 events, letter n is in envelope n, $n = 1, 2, 3, 4$. The probability of at least one event occurring is $1/4$. The probability of two events is $(1/4)(1/3) = 1/12$. The probability of three events is $\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{24}$. And that is also the probability of all four events occurring. There are 6 ways to select 2 envelopes and 4 ways to select 3 envelopes.

The probability of at least one event occurring is

$$4\left(\frac{1}{4}\right) - 6\left(\frac{1}{12}\right) + 4\left(\frac{1}{24}\right) - \frac{1}{24} = \boxed{\frac{5}{8}} \qquad \textbf{(D)}$$

1.34. The probability of $U \cup V$ cannot be more than 1, and

$$P[U \cap V] = P[U] + P[V] - P[U \cup V] = 0.8 + 0.4 - P[U \cup V]$$

Using the maximal value of $P[U \cup V]$, we get the minimal value of $P[U \cap V] \geq 0.8 + 0.4 - 1 = \boxed{\mathbf{0.2}}$.

Practice Exam 1

1. Six actuarial students are all equally likely to pass Exam P. Their probabilities of passing are mutually independent. The probability all 6 pass is 0.24.

Calculate the variance of the number of students who pass.

(A) 0.98 (B) 1.00 (C) 1.02 (D) 1.04 (E) 1.06

2. The number of claims in a year, N, on an insurance policy has a Poisson distribution with mean 0.25. The numbers of claims in different years are mutually independent.

Calculate the probability of 3 or more claims over a period of 2 years.

(A) 0.001 (B) 0.010 (C) 0.012 (D) 0.014 (E) 0.016

3. Claim sizes on an insurance policy have the following distribution:

$$F(x) = \begin{cases} 0, & x \le 0 \\ 0.0002x & 0 < x < 1000 \\ 0.4, & x = 1000 \\ 1 - 0.6e^{-(x-1000)/2000} & x > 1000 \end{cases}$$

Calculate expected claim size.

(A) 1500 (B) 1700 (C) 1900 (D) 2100 (E) 2300

4. An actuary analyzes weekly sales of automobile insurance, X, and homeowners insurance, Y. The analysis reveals that $\operatorname{Var}(X) = 2500$, $\operatorname{Var}(Y) = 900$, and $\operatorname{Var}(X + Y) = 3100$.

Calculate the correlation of X and Y.

(A) -0.2 (B) -0.1 (C) 0 (D) 0.1 (E) 0.2

5. A device runs until either of two components fail, at which point the device stops running. The joint distribution function of the lifetimes of the two components is $F(s, t)$. The joint density function is nonzero only when $0 < s < 1$ and $0 < t < 1$.

Determine which of the following represents the probability that the device fails during the first half hour of operation.

(A) $F(0.5, 0.5)$
(B) $1 - F(0.5, 0.5)$
(C) $F(0.5, 1) + F(1, 0.5)$
(D) $F(0.5, 1) + F(1, 0.5) - F(0.5, 0.5)$
(E) $F(0.5, 1) + F(1, 0.5) - F(1, 1)$

6. The probability of rain each day is the same, and occurrences of rain are mutually independent.

The expected number of non-rainy days before the next rain is 4.

Calculate the probability that the second rain will not occur before 7 non-rainy days.

(A) 0.11 (B) 0.23 (C) 0.40 (D) 0.50 (E) 0.55

Exam questions continue on the next page . . .

7. On a certain day, you have a staff meeting and an actuarial training class. Time in hours for the staff meeting is X and time in hours for the actuarial training session is Y. X and Y have the joint density function

$$f(x,y) = \begin{cases} \dfrac{3x+y}{250} & 0 \le x \le 5,\ 0 \le y \le 5 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the expected total hours spent in the staff meeting and actuarial training class.

(A) 3.33 (B) 4.25 (C) 4.67 (D) 5.17 (E) 5.83

8. In a small metropolitan area, annual losses due to storm and fire are assumed to be independent, exponentially distributed random variables with respective means 1.0 and 2.0.

Calculate the expected value of the maximum of these losses.

(A) 2.33 (B) 2.44 (C) 2.56 (D) 2.67 (E) 2.78

9. A continuous random variable X has the following distribution function:

$$F(x) = \begin{cases} 0 & x \le 0 \\ 0.2 & x = 3 \\ 0.4 & x = 8 \\ 0.7 & x = 16 \\ 1 & x \ge 34 \end{cases}$$

For $0 < x < 34$ not specified, $F(x)$ is determined by linear interpolation between the nearest two specified values.

Calculate the 80^{th} percentile of X.

(A) 20 (B) 22 (C) 24 (D) 26 (E) 28

10. The side of a cube is measured with a ruler. The error in the measurement is uniformly distributed on $[-0.2, 0.2]$.

The measurement is 4.

Calculate the expected volume of the cube.

(A) 63.84 (B) 63.92 (C) 64.00 (D) 64.08 (E) 64.16

11. The amount of time at the motor vehicles office to renew a license is modeled with two random variables. X represents the time in minutes waiting in line, and Y represents the total time in minutes including both time in line and processing time. The joint density function of X and Y is

$$f(x,y) = \begin{cases} \dfrac{3}{5000} e^{-(x+2y)/100} & 0 < x < y \\ 0, & \text{otherwise} \end{cases}$$

Calculate the probability that the time in line is less than 30 minutes, given that the total time in the office is 50 minutes.

(A) 0.54 (B) 0.57 (C) 0.60 (D) 0.63 (E) 0.66

Exam questions continue on the next page . . .

12. Let X and Y be independent Bernoulli random variables with $p = 0.5$.

Determine the moment generating function of $X - Y$.

(A) $\dfrac{0.5 + 0.5e^t}{0.5 - 0.5e^t}$

(B) $(0.5 + 0.5e^t)^2$

(C) $(0.5 + 0.5e^t)(0.5 - 0.5e^t)$

(D) $e^{-t}(0.5 + 0.5e^t)^2$

(E) $e^{-t}(0.5 + 0.5e^t)(0.5 + 0.5e^{-t})$

13. The quality of drivers is measured by a random variable Θ. This variable is uniformly distributed on $[0, 1]$.

Given a driver of quality $\Theta = \theta$, annual claims costs for that driver have the following distribution function:

$$F(x \mid \theta) = \begin{cases} 0 & x \le 100 \\ 1 - \left(\dfrac{100}{x}\right)^{2+\theta} & x > 100 \end{cases}$$

Calculate the expected annual claim costs for a randomly selected driver.

(A) 165 (B) 167 (C) 169 (D) 171 (E) 173

14. Losses on an auto liability policy due to bodily injury (X) and property damage (Y) follow a bivariate normal distribution, with

$$\mathbf{E}[X] = 150 \qquad \mathbf{E}[Y] = 50$$
$$\operatorname{Var}(X) = 5{,}000 \qquad \operatorname{Var}(Y) = 400$$

and correlation coefficient 0.8.

Calculate the probability that total losses, $X + Y$, are less than 250.

(A) 0.571 (B) 0.626 (C) 0.680 (D) 0.716 (E) 0.751

15. On an insurance coverage, the number of claims submitted has the following probability function:

Number of claims	Probability
0	0.30
1	0.25
2	0.25
3	0.20

The size of each claim has the following probability function:

Size of claim	Probability
10	0.5
20	0.3
30	0.2

Claim sizes are independent of each other and of claim counts.

Calculate the mode of the sum of claims.

(A) 0 (B) 10 (C) 20 (D) 30 (E) 40

Exam questions continue on the next page . . .

16. Among commuters to work:

(i) 62% use a train.
(ii) 25% use a bus.
(iii) 18% use a car.
(iv) 16% use a train and a bus.
(v) 10% use a train and a car.
(vi) 8% use a bus and a car.
(vii) 2% use a train, a bus, and a car.

Calculate the proportion of commuters who do not use a train, a bus, or a car.

(A) 0.25 (B) 0.27 (C) 0.29 (D) 0.31 (E) 0.33

17. Two fair dice are tossed.

Calculate the probability that the numbers of the dice are even, given that their sum is 6.

(A) 1/3 (B) 2/5 (C) 1/2 (D) 3/5 (E) 2/3

18. In a box of 100 machine parts, 6 are defective. 5 parts are selected at random.

Calculate the probability that exactly 4 selected parts are not defective.

(A) 0.05 (B) 0.19 (C) 0.24 (D) 0.28 (E) 0.31

19. Let X and Y be discrete random variables with joint probability function

$$p(x,y) = \begin{cases} \dfrac{x+y}{18} & (x,y) = (1,1),(1,2),(2,1),(2,4),(3,1) \\ 0, & \text{otherwise} \end{cases}$$

Determine the marginal probability function of Y.

(A) $p(x) = \begin{cases} \dfrac{5}{18} & y=1 \\ \dfrac{1}{2} & y=2 \\ \dfrac{2}{9} & y=3 \end{cases}$

(B) $p(x) = \begin{cases} \dfrac{5}{18} & y=1 \\ \dfrac{1}{2} & y=2 \\ \dfrac{2}{9} & y=4 \end{cases}$

(C) $p(x) = \begin{cases} \dfrac{1}{2} & y=1 \\ \dfrac{1}{6} & y=2 \\ \dfrac{1}{3} & y=4 \end{cases}$

(D) $p(x) = \begin{cases} \dfrac{3}{5} & y=1 \\ \dfrac{1}{5} & y=2 \\ \dfrac{1}{5} & y=4 \end{cases}$

(E) $p(x) = \begin{cases} \dfrac{2}{3} & y=1 \\ \dfrac{1}{9} & y=2 \\ \dfrac{2}{9} & y=4 \end{cases}$

Exam questions continue on the next page . . .

20. A blood test for a disease detects the disease if it is present with probability 0.95. If the disease is not present, the test produces a false positive for the disease with probability 0.03.

2% of a population has this disease.

Calculate the probability that a randomly selected individual has the disease, given that the blood test is positive.

(A) 0.05 (B) 0.39 (C) 0.54 (D) 0.73 (E) 0.97

21. The trip to work involves:

- Driving to the train station. Let X be driving time. Mean time is 10 minutes and standard deviation is 5 minutes.
- Taking the train. Let Y be train time. Mean time is 35 minutes with standard deviation 60 minutes.

Based on a sample of 100 trips, the 67$^{\text{th}}$ percentile of time for the trip is 47.693 minutes.

Using the normal approximation, determine the approximate correlation factor between X and Y, ρ_{XY}.

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4 (E) 0.5

22. An actuarial department has 8 pre-ASA students and 5 students who are ASAs. To support a project to convert reserves to a new software system, the head of the project selects 4 students randomly.

Calculate the probability that at least 2 ASAs were selected.

(A) 0.39 (B) 0.46 (C) 0.51 (D) 0.56 (E) 0.63

23. The amount of time a battery lasts, T, is normally distributed. The 20$^{\text{th}}$ percentile of T is 160 and the 30$^{\text{th}}$ percentile is 185.

Calculate the 60$^{\text{th}}$ percentile of T.

(A) 246 (B) 253 (C) 260 (D) 267 (E) 274

24. An index for the cost of automobile replacement parts, X, is exponentially distributed with mean 10. Given that $X = x$, the cost of repairing a car, Y, is exponentially distributed with mean X.

Calculate $\operatorname{Cov}(X, Y)$.

(A) 50 (B) 100 (C) 141 (D) 173 (E) 200

25. The side of a square is measured. The true length of the side is 10. The length recorded by the measuring instrument is normally distributed with mean 10 and standard deviation 0.1.

Calculate the expected area of the square based on the measurement recorded by the measuring instrument.

(A) 99.90 (B) 99.99 (C) 100.00 (D) 100.01 (E) 100.10

26. Losses on an insurance policy are modeled with a random variable with density function

$$f(x) = \begin{cases} cx^a & 0 < x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

The probability that a loss is less than 2, given that it is less than 3, is 0.5227.

Calculate the probability that a loss is greater than 1, given that it is less than 2.

(A) 0.64 (B) 0.67 (C) 0.70 (D) 0.73 (E) 0.76

Exam questions continue on the next page . . .

27. Let X be a random variable with the following distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \le x < 1 \\ 0.3 & 1 \le x < 2 \\ 0.5 + 0.1x & 2 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

Calculate $P[1 \le X \le 2]$.

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4 (E) 0.5

28. X and Y are random variables with joint density function

$$f(x, y) = \begin{cases} 0.5(|x| + y) & -1 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Calculate $\mathbf{E}[X^4 Y]$.

(A) 0.05 (B) 0.08 (C) 0.10 (D) 0.12 (E) 0.15

29. A pair of dice is tossed.
Calculate the variance of the sum of the dice.

(A) $\frac{17}{3}$ (B) $\frac{35}{6}$ (C) 6 (D) $\frac{37}{6}$ (E) $\frac{19}{3}$

30. The daily number of visitors to a national park follows a Poisson distribution with mean 900.
Calculate the 80^{th} percentile of the number of visitors in a day.

(A) 915 (B) 920 (C) 925 (D) 930 (E) 935

Solutions to the above questions begin on page 417.

Appendix A. Solutions to the Practice Exams

Answer Key for Practice Exam 1

1	B	6	D	11	E	16	B	21	B	26	B
2	D	7	E	12	D	17	B	22	C	27	E
3	D	8	A	13	C	18	C	23	A	28	E
4	B	9	B	14	D	19	C	24	B	29	B
5	D	10	E	15	A	20	B	25	D	30	C

Practice Exam 1

1. [**Lesson 19**]

$$\begin{aligned} p^6 &= 0.24 \\ \mathrm{Var}(N) &= 6p(1-p) = 6\sqrt[6]{0.24}(1-\sqrt[6]{0.24}) = \boxed{\mathbf{1.0012}} \qquad \textbf{(B)} \end{aligned}$$

2. [**Lesson 21**] Over two years, the Poisson parameter is 2(0.25) = 0.5.

$$\Pr(N>2) = 1-p(0)-p(1)-p(2) = 1-e^{-0.5}\left(1+0.5+\frac{0.5^2}{2}\right) = \boxed{\mathbf{0.01439}} \qquad \textbf{(D)}$$

3. [**Lesson 7**] We will integrate the survival function.

$$\begin{aligned} \int_0^{1000}\bigl(1-F(x)\bigr)\,\mathrm{d}x &= \int_0^{1000}(1-0.0002x)\,\mathrm{d}x \\ &= 1000-0.0001(1000^2) = 900 \\ \int_{1000}^{\infty}\bigl(1-F(x)\bigr)\,\mathrm{d}x &= \int_{1000}^{\infty}0.6e^{-(x-1000)/2000}\,\mathrm{d}x \\ &= 0.6(2000) = 1200 \\ \mathbf{E}[X] &= 900+1200 = \boxed{\mathbf{2100}} \end{aligned}$$

You can also do this using the double expectation formula. Given that $X<1000$, it is uniform on [0,1000] with mean 500. Given that it is greater than 1000, the excess over 1000 is exponential with mean 2000, so the total mean is 1000 + 2000=3000.

$$\begin{aligned} \Pr(X<1000) &= 0.2 \\ \Pr(X=1000) &= 0.2 \\ \Pr(X>1000) &= 0.6 \\ \mathbf{E}[X] &= \Pr(X<1000)(500)+\Pr(X=1000)(1000)+\Pr(X>1000)(3000) \\ &= 0.2(500)+0.2(1000)+0.6(3000) = \boxed{\mathbf{2100}} \qquad \textbf{(D)} \end{aligned}$$

4. **[Lesson 15]** $3100 = 2500 + 900 + 2\operatorname{Cov}(X, Y)$, so $\operatorname{Cov}(X, Y) = -150$. Then

$$\operatorname{Corr}(X, Y) = \frac{-150}{\sqrt{(2500)(900)}} = \boxed{-0.1} \qquad \textbf{(B)}$$

5. **[Lesson 11]** If A is the failure of the first component and B the failure of the second component, we want

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

and **(D)** is exactly that.

6. **[Lesson 20]** Let p be the probability of rain. The negative binomial random variable for the first rain, with $k = 1$, has mean 4, so $(1 - p)/p = 4$ and $p = 0.2$. We want the probability of less than 2 rainy days in the next 8 days. That probability is

$$\binom{8}{0}0.8^8 + \binom{8}{1}(0.2)(0.8^7) = \boxed{0.5033} \qquad \textbf{(D)}$$

7. **[Lesson 14]**

$$\begin{aligned}
\mathbf{E}[X + Y] &= \frac{1}{250}\int_0^5\int_0^5 (x + y)(3x + y)\,\mathrm{d}y\,\mathrm{d}x \\
&= \frac{1}{250}\int_0^5\int_0^5 (3x^2 + 4xy + y^2)\,\mathrm{d}y\,\mathrm{d}x \\
&= \frac{1}{250}\int_0^5 \left(3x^2y + 2xy^2 + \frac{y^3}{3}\right)\Bigg|_0^5 \mathrm{d}x \\
&= \frac{1}{250}\int_0^5 \left(15x^2 + 50x + \frac{125}{3}\right)\mathrm{d}x \\
&= \frac{1}{250}\left(5x^3 + 25x^2 + \frac{125}{3}x\right)\Bigg|_0^5 \\
&= \frac{1}{250}\left(625 + 625 + \frac{625}{3}\right) = \boxed{5.83333} \qquad \textbf{(E)}
\end{aligned}$$

8. **[Lesson 26]** Let Y be the maximum. The distribution function of Y for $x > 0$ is

$$\begin{aligned}
F_Y(x) &= (1 - e^{-x})(1 - e^{-x/2}) \\
&= 1 - e^{-x} - e^{-x/2} + e^{-3x/2}
\end{aligned}$$

The density function of Y is

$$f_Y(x) = e^{-x} + 0.5e^{-x/2} - 1.5e^{-3x/2}$$

Use (or prove) the fact that $\int_0^\infty xe^{-cx} = 1/c^2$. The expected value of Y is

$$\begin{aligned}
\mathbf{E}[Y] &= \int_0^\infty \left(xe^{-x} + 0.5xe^{-x/2} - 1.5xe^{-3x/2}\right)\mathrm{d}x \\
&= 1 + 2 - \frac{2}{3} = \boxed{2.3333} \qquad \textbf{(A)}
\end{aligned}$$

A faster way to get the answer is to note that the expected value of the sum is $1 + 2 = 3$. The minimum is exponential with mean $1/(1 + 1/2) = 2/3$. So the maximum's mean must be $3 - 2/3$.

9. **[Lesson 9]** We interpolate linearly between $x = 16$ and $x = 34$ to find x such that $F(x) = 0.8$.

$$16 + \frac{0.8 - 0.7}{1 - 0.7}(34 - 16) = \boxed{\mathbf{22}} \qquad \textbf{(B)}$$

10. **[Lesson 8]** The size of the side is a random variable X uniform on $[3.8, 4.2]$. The density is $1/0.4$. We want $\mathbf{E}[X^3]$.

$$\begin{aligned}\mathbf{E}[X^3] &= \int_{3.8}^{4.2} \frac{x^3\,dx}{0.4}\\ &= \left.\frac{x^4}{4(0.4)}\right|_{3.8}^{4.2}\\ &= \frac{4.2^4 - 3.8^4}{1.6} = \boxed{\mathbf{64.16}} \qquad \textbf{(E)}\end{aligned}$$

11. **[Lesson 16]** The numerator of the fraction for the probability we seek is the integral of the joint density function over $X < 30$, $Y = 50$. The denominator is the integral of the joint density function over all X with $Y = 50$, but $X < Y$. The $3/5000$ will cancel out in the fraction, so we'll ignore it.

$$\begin{aligned}\int_0^{50} e^{-(x+100)/100}\,dx &= -\left.100e^{-(x+100)/100}\right|_0^{50}\\ &= 100(e^{-1} - e^{-1.5})\\ \int_0^{30} e^{-(x+100)/100}\,dx &= -\left.100e^{-(x+100)/100}\right|_0^{30}\\ &= 100(e^{-1} - e^{-1.3})\\ \Pr(X < 30 \mid Y = 50) &= \frac{e^{-1} - e^{-1.3}}{e^{-1} - e^{-1.5}} = \boxed{\mathbf{0.6587}} \qquad \textbf{(E)}\end{aligned}$$

12. **[Lesson 27]** The probability function of $X - Y$ is

n	$p(n)$
−1	0.25
0	0.50
1	0.25

Therefore, the MGF is

$$M(t) = 0.25e^{-t} + 0.5 + 0.25e^{t}$$

which is the same as **(D)**

13. **[Lesson 18]** We'll use the double expectation formula. The density function of the conditional claim costs is

$$f(x \mid \theta) = \frac{dF(x \mid \theta)}{dx} = \frac{(2+\theta)100^{2+\theta}}{x^{3+\theta}}$$

with mean

$$\mathbf{E}[X \mid \theta] = \int_{100}^{\infty} \frac{(2+\theta)100^{2+\theta}\,dx}{x^{2+\theta}} = \frac{100(2+\theta)}{(1+\theta)}$$

We now use the double expectation formula.

$$\begin{aligned}\mathbf{E}[X] &= \mathbf{E}_\Theta\big[\mathbf{E}[X \mid \theta]\big]\\ &= \int_0^1 \frac{100(2+\theta)}{1+\theta} f_\Theta(\theta)\mathrm{d}\theta\end{aligned}$$

Θ follows a uniform distribution on $[0,1]$, so $f_\Theta(\theta) = 1$.

$$\begin{aligned}\mathbf{E}[X] &= \int_0^1 \frac{100(2+\theta)}{1+\theta}\mathrm{d}\theta\\ &= 100\int_0^1 \left(1+\frac{1}{1+\theta}\right)\mathrm{d}\theta\\ &= 100\big(\theta+\ln(1+\theta)\big)\Big|_0^1\\ &= 100(1+\ln 2) = \boxed{\mathbf{169.31}} \qquad \textbf{(C)}\end{aligned}$$

14. **[Lesson 24]** The sum is normal with mean $150 + 50 = 200$ and variance

$$5000 + 400 + 2(0.8)\sqrt{(5000)(400)} = 7662.74$$

The probability this is less than 250 is

$$\Phi\left(\frac{250-200}{\sqrt{7662.74}}\right) = \Phi(0.571) = \boxed{\mathbf{0.7160}} \qquad \textbf{(D)}$$

15. **[Lesson 10]** Let X be total claims. We have to add up probabilities of all ways of reaching multiples of 10.

$$\begin{aligned}\Pr(X=0) &= 0.3\\ \Pr(X=10) &= 0.25(0.5) = 0.125\\ \Pr(X=20) &= 0.25(0.3) + 0.25(0.5^2) = 0.1375\\ \Pr(X=30) &= 0.25(0.2) + 0.25(2)(0.5)(0.3) + 0.20(0.5^3) = 0.15\end{aligned}$$

We stop here, because the probabilities already sum up to 0.7125, so the remaining probabilities are certainly less than 0.3, the probability of $\boxed{0}$. **(A)**

16. **[Lesson 1]** Let T be train, B be bus, C be car.

$$\begin{aligned}P[T\cup B\cup C] &= P[T]+P[B]+P[C]-P[T\cap B]-P[T\cap C]-P[B\cap C]+P[T\cap B\cap C]\\ &= 0.62+0.25+0.18-0.16-0.10-0.08+0.02 = 0.73\\ P[(T\cup B\cup C)'] &= 1-0.73 = \boxed{\mathbf{0.27}} \qquad \textbf{(B)}\end{aligned}$$

17. **[Lesson 3]** There are three ways to get 6 as a sum of two odd numbers: $1+5, 3+3, 5+1$. There are two ways to get 6 as a sum of two even numbers: $2+4$ and $4+2$. Since these are all equally likely, the probability that both are even is $\boxed{2/5}$. **(B)**

18. **[Lesson 2]**

$$\frac{\binom{94}{4}\binom{6}{1}}{\binom{100}{5}} = \boxed{\mathbf{0.24303}} \qquad \textbf{(C)}$$

19. **[Lesson 13]** $\Pr(Y = 1)$ is the sum of the probabilities of (1,1), (2,1), (3,1), or

$$\frac{1+1+2+1+3+1}{18} = \frac{1}{2}$$

We already see that **(C)** is the answer. Continuing, $\Pr(Y = 2)$ is the probability of (1,2), or $(1 + 2)/18 = 1/6$, and $\Pr(Y = 4)$ is the probability of (2,4), or $(2 + 4)/18 = 1/3$.

20. **[Lesson 4]** Use Bayes' Theorem. Let D be the disease, P a positive result of the test.

$$P[D \mid P] = \frac{P[P \mid D]P[D]}{P[P \mid D]P[D] + P[P \mid D']P[D']}$$
$$= \frac{(0.95)(0.02)}{(0.95)(0.02) + (0.03)(0.98)} = \boxed{\mathbf{0.3926}} \qquad \textbf{(B)}$$

21. **[Lesson 25]** The mean time for the trip is $10 + 35 = 45$. Let $Z = X + Y$ and let $\bar{Z}$ be the sample mean of Z. Based on the normal approximation applied to the given information,

$$45 + z_{0.67}\sqrt{\operatorname{Var}(\bar{Z})} = 45 + 0.44\sqrt{\operatorname{Var}(\bar{Z})} = 47.693$$
$$\operatorname{Var}(\bar{Z}) = \left(\frac{2.693}{0.44}\right)^2 = 37.460$$

The variance of the mean is the variance of the distribution divided by the size of the sample, so the variance of Z is approximately 3746.0. Back out $\operatorname{Cov}(X, Y)$:

$$3746.0 = 5^2 + 60^2 + 2\operatorname{Cov}(X, Y)$$
$$\operatorname{Cov}(X, Y) = \frac{3746 - 3625}{2} = 60.50$$

The correlation coefficient is approximately

$$\rho = \frac{60.50}{(5)(60)} = \boxed{\mathbf{0.202}} \qquad \textbf{(B)}$$

22. **[Lesson 2]** Total number of selections: $\binom{13}{4} = 715$.
Ways to select 2 ASAs: $\binom{8}{2}\binom{5}{2} = 280$.
Ways to select 3 ASAs: $\binom{8}{1}\binom{5}{3} = 80$.
Ways to select 4 ASAs: $\binom{8}{0}\binom{5}{4} = 5$.

$$\frac{280 + 80 + 5}{715} = \boxed{\mathbf{0.51049}} \qquad \textbf{(C)}$$

23. **[Lesson 23]** For a standard normal distribution, 20^{th} percentile is -0.842 and 30^{th} percentile is -0.524. Also, 60^{th} percentile is 0.253. We have

$$\mu - 0.842\sigma = 160$$
$$\mu - 0.524\sigma = 185$$

So

$$\sigma = \frac{185 - 160}{0.842 - 0.524} = 78.616$$

and

$$\mu + 0.253\sigma = 185 + (0.253 + 0.524)(78.616) = \boxed{\mathbf{246}} \qquad \textbf{(A)}$$

24. **[Lesson 22]** $\operatorname{Cov}(X,Y) = \mathbf{E}[XY] - \mathbf{E}[X]\,\mathbf{E}[Y]$ and $\mathbf{E}[X] = 10$. The expected value of Y can be calculated using the double expectation formula:

$$\mathbf{E}[Y] = \mathbf{E}\big[\mathbf{E}[Y \mid X]\big] = \mathbf{E}[X] = 10$$

To calculate $\mathbf{E}[XY]$, use the double expectation formula.

$$\mathbf{E}[XY] = \mathbf{E}\big[\mathbf{E}[XY \mid X]\big] = \mathbf{E}[X^2]$$

The second moment of an exponential is $\operatorname{Var}(X) + \mathbf{E}[X]^2$, which here is $100 + 10^2 = 200$. Then

$$\operatorname{Cov}(X,Y) = 200 - (10)(10) = \boxed{\mathbf{100}} \qquad \textbf{(B)}$$

25. **[Lesson 23]** We are asked for $\mathbf{E}[X^2]$ where X is the length recorded by the measuring instrument.

$$\mathbf{E}[X^2] = \operatorname{Var}(X) + \mathbf{E}[X]^2 = 0.1^2 + 10^2 = \boxed{\mathbf{100.01}} \qquad \textbf{(D)}$$

26. **[Lesson 6]** Let's solve for a.

$$\begin{aligned}
0.5227 = \Pr(X < 2 \mid X < 3) &= \frac{F(2)}{F(3)} \\
&= \frac{2^{a+1}}{3^{a+1}} = \left(\frac{2}{3}\right)^{a+1} \\
\ln 0.5227 &= (a+1)\ln(2/3) \\
a &= \frac{\ln 0.5227}{\ln(2/3)} - 1 = 0.6
\end{aligned}$$

Now we can calculate $\Pr(X > 1 \mid X < 2)$. We never need c, since it cancels in numerator and denominator.

$$\begin{aligned}
\Pr(X > 1 \mid X < 2) &= \frac{\Pr(1 < X < 2)}{\Pr(X < 2)} \\
&= \frac{2^{a+1} - 1^{a+1}}{2^{a+1}} \\
&= \frac{2^{1.6} - 1}{2^{1.6}} = \boxed{\mathbf{0.6701}} \qquad \textbf{(B)}
\end{aligned}$$

27. **[Lesson 5]**

$$F(2) - F(1^-) = 0.7 - 0.2 = \boxed{\mathbf{0.5}} \qquad \textbf{(E)}$$

28. **[Lesson 14]** Notice that the density and X^4Y are symmetric around $x = 0$, so we can calculate the required integral from $x = 0$ to $x = 1$ and double it.

$$\begin{aligned}
0.5\,\mathbf{E}[X^4Y] &= \int_0^1\int_0^1 0.5x^4y(x+y)\,\mathrm{d}y\,\mathrm{d}x \\
&= 0.5\int_0^1\int_0^1 (x^5y + x^4y^2)\mathrm{d}y\,\mathrm{d}x \\
&= 0.5\int_0^1 \left(\frac{x^5y^2}{2} + \frac{x^4y^3}{3}\right)\bigg|_0^1 \mathrm{d}x \\
&= 0.5\int_0^1 \left(\frac{x^5}{2} + \frac{x^4}{3}\right)\mathrm{d}x \\
&= 0.5\left(\frac{1}{12} + \frac{1}{15}\right) = 0.5(0.15)
\end{aligned}$$

The answer is **0.15**. **(E)**

29. **[Lesson 12]** Since the dice are independent, the variance of the sum is the sum of the variances. The variance of each die's toss is $(n^2 - 1)/12$ with $n = 6$, or $35/12$. The variance of the sum of two dice is **35/6**. **(B)**

30. **[Lesson 25]** It is not reasonable to calculate this exactly, so the normal approximation is used. The 80th percentile of a standard normal distribution is 0.842. The mean and variance of number of visitors is 900. So the 80th percentile of number of visitors is $900 + 0.842\sqrt{900} =$ **925.26**. **(C)**