Solutions to EA-1 Examination
Spring, 2005

Question 1

The dollar-weighted rate of return for the fund (for $i = 6.00\%$) is:

$$1{,}700 = (1{,}000)(1 + i) + (C_1)(1 + i) + C_2 - (100)(1 + \tfrac{9}{12} i) - 150$$
$$1{,}700 = (1{,}000)(1.06) + (C_1)(1.06) + C_2 - (100)(1.045) - 150$$
$$894.50 = 1.06C_1 + C_2 \qquad \Rightarrow \qquad C_2 = 894.50 - 1.06C_1$$

The time-weighted rate of return is:

$$1.0625 = \frac{\text{Account value on } 3/31/2005^*}{\text{Account value on } 1/1/2005^{**}} \times \frac{\text{Account value on } 12/31/2005^*}{\text{Account value on } 4/1/2005^{**}}$$

$$= ((1{,}300 + 100)/(1{,}000 + C_1)) \times ((1{,}700 - C_2 + 150)/(1{,}300))$$

$$.986607 = (1850 - C_2)/(1{,}000 + C_1)$$
$$.986607 \times (1{,}000 + C_1) = 1{,}850 - (894.50 - 1.06C_1) = 955.55 + 1.06C_1$$
$$C_1 = 423$$

* Before contributions or withdrawals
** After contributions or withdrawals

Answer is B.

Question 2

For $i = .05$, $i^{(2)}/2 = 1.05^{1/2} - 1 = .024695$

Since there are 10 bonds, the face amount of each bond is \$10,000 (\$100,000/10). The bond coupons are payable semiannually, each coupon equal to \$300 (3% of \$10,000). Taking the present value at the beginning of the year of each year's coupons:

$300\, a_{\overline{2}|.024695} = 578$

The redemption value of each bond is \$12,500 (125% of \$10,000), with the exception of the 4th redemption (equal to \$10,000). The present value of the redemption values is:

$12{,}500\, a_{\overline{10}|.05}\, v^9 - 2{,}500 v^{13} = 60{,}893$

Since the first bond isn't redeemed until the end of the 10th year, there will be coupon payments for all 10 bonds in the amount of \$5,780 (578×10) for each of the first 10 years. Beginning in the 11th year, the annual coupon payments will reduce by \$578 each year. The present value of the coupon payments is:

$5{,}780\, \ddot{a}_{\overline{10}|.05} + 578\,(Da)_{\overline{9}|.05}\, v^9 = 60{,}963$

The purchase price of the serial bond is equal to the sum of the present value of the redemption value and the present value of the coupon payments:

$60{,}893 + 60{,}963 = 121{,}856$

Answer is B.

Question 3

For $i^{(2)} = .05$, $i = 1.025^2 - 1 = .050625$

Annual loan payment = \$1,000/ $a_{\overline{20}|.050625}$ = \$80.67

Accumulated loan payments after 20 years = \$80.67 $s_{\overline{20}|.06}$ = \$2,967

$1 + R = (2{,}967/1{,}000)^{1/20} = 1.0559$
$R = 5.59\%$

Answer is B.

Question 4

The present value of each annuity can be written as follows:

I. $100a_{65}$
II. $90a_{65:65} + (54a_{65|65}) \times 2 = 90a_{65:65} + (54(a_{65} - a_{65:65}) \times 2) = 108a_{65} - 18a_{65:65}$
III. $70a_{65:65} + (P(a_{65} - a_{65:65}) \times 2) = 2Pa_{65} + (70 - 2P)a_{65:65}$

Setting the first two actuarially equivalent annuities equal,

$100a_{65} = 108a_{65} - 18a_{65:65}$
$100 = 108 - 18 \times (a_{65:65}/a_{65})$
$a_{65:65}/a_{65} = 0.4444$

Setting the first and third actuarially equivalent annuities equal,

$100a_{65} = 2Pa_{65} + (70 - 2P)a_{65:65}$
$100 = 2P + (70 - 2P)(a_{65:65}/a_{65}) = 2P + (70 - 2P)(0.4444)$
$P = 62$

Answer is D.

Question 5

The payments under the terms of the annuity are guaranteed for the first 5 years, and the payments after the first 5 years are made only if Smith was still alive 5 years earlier. Using first principles to write an expression for the present value of the annuity:

$$\begin{aligned}
PV &= 1{,}000\, a_{\overline{5}|} + 1{,}000(v^6\, p_{65} + v^7\, {}_2p_{65} + v^8\, {}_3p_{65} + \ldots) \\
&= 1{,}000\, a_{\overline{5}|} + 1{,}000v^5(v\, p_{65} + v^2\, {}_2p_{65} + v^3\, {}_3p_{65} + \ldots) \\
&= 1{,}000\, a_{\overline{5}|} + 1{,}000v^5 a_{65} \\
&= 4{,}329 + 7{,}973 \\
&= 12{,}302
\end{aligned}$$

Answer is C.

Question 6

Since mortality rates double beginning at age 75, the mortality rate at age 75 and greater is 0.10, and the probability of survival at age 75 and greater is 0.90.

$$\begin{aligned}
Y &= 1{,}000\, a_{65:\overline{25}|} = 1{,}000 \times (v\, p_{65} + v^2\, {}_2p_{65} + \ldots + v^{25}\, {}_{25}p_{65}) \\
&= 1{,}000 \times (v\, p_{65} + v^2\, {}_2p_{65} + \ldots + v^{10}\, {}_{10}p_{65} + v^{11}\, {}_{10}p_{65}\, p_{75} + \ldots + v^{25}\, {}_{10}p_{65}\, {}_{15}p_{75}) \\
&= 1{,}000 \times (.95v + (.95v)^2 + \ldots + (.95v)^{10} + (.95v)^{10}(.9v) + \ldots + (.95v)^{10}(.9v)^{15}) \\
&= 1{,}000 \times (.95v + (.95v)^2 + \ldots + (.95v)^{10} + (.95v)^{10}[(.9v) + \ldots + (.9v)^{15}])
\end{aligned}$$

Note that $.95v = .95/1.05 = 1/(1.05/.95) = 1/1.1053$,
and $.9v = .9/1.05 = 1/(1.05/.9) = 1/1.1667$

Therefore, $Y = 1{,}000 \times (a_{\overline{10}|.1053} + (.95v_{.05})^{10}(a_{\overline{15}|.1667})) = 7{,}994$

Answer is C.

Question 7

$p_x = l_{x+1}/l_x = Bc^{-(x+1)}/Bc^{-x} = 1.01^{-(x+1)}/1.01^{-x} = 1.01^{-1} = 0.990099$ for all $x > 0$

$p_{xx} = (p_x)(p_x) = 0.990099^2 = 0.980296$

Let the single premium be P.

$P = 1{,}000\, a_{\overline{xx}} = 1{,}000\ (a_x + a_x - a_{xx})$
$\quad = 1{,}000 \times [(v\, p_x + v^2\ {}_2p_x + v^3\ {}_3p_x + \ldots) \times 2 - (v\, p_{xx} + v^2\ {}_2p_{xx} + v^3\ {}_3p_{xx} + \ldots)]$
$\quad = 1{,}000 \times [(.990099v + (.990099v)^2 + \ldots) \times 2 - (.980296v + (.980296v)^2 + \ldots)]$

Note that $.990099v = .990099/1.05 = 1/(1.05/.990099) = 1/1.0605$,
and $.980296v = .\ 980296/1.05 = 1/(1.05/.980296) = 1/1.0711$

Therefore, $P = 1{,}000 \times (2\, a_{\overline{\infty}|.0605} - a_{\overline{\infty}|.0711}) = 1{,}000 \times ((2/.0605) - 1/.0711)$
$\quad = 18{,}993$

Answer is A.

Question 8

${}_{n|}a_{\overline{2n|}} = v^{n+1} + v^{n+2} + \ldots + v^{3n} = 8.00407$
${}_{n-1|}a_{\overline{2n+1|}} = v^{n} + v^{n+1} + \ldots + v^{3n} = 8.63279$
${}_{n-1|}a_{\overline{2n+1|}} - {}_{n|}a_{\overline{2n|}} = v^{n} = 8.63279 - 8.00407 = 0.62872$

${}_{n|}a_{\overline{2n|}} = v^{n} a_{\overline{2n|}} = v^{n} \left(\frac{1 - v^{2n}}{i}\right) = 0.62872 \times \left(\frac{1 - 0.62872^{2}}{i}\right) = 8.00407$
$i = .0475$, or 4.75%

Answer is A.

Question 9

Note that $p_{40} = l_{41}/l_{40} = \frac{59}{60}$, $p_{41} = l_{42}/l_{41} = \frac{58}{59}$, etc.
Also, the last age at which there is a possibility of an annuity payment is age 99. That means that there is a maximum of 59 annuity payments.

Let the single premium be P.

$$\begin{aligned} P &= 100{,}000 \times (q_{40}q_{40}v + p_{40}p_{40}q_{41}q_{41}v^2 + \ldots) \\ &= 100{,}000 \times \left(\left(\tfrac{1}{60}\right)^2 v + \left(\tfrac{59}{60} \times \tfrac{1}{59}\right)^2 v^2 + \ldots \right) \\ &= 100{,}000 \times \left(\tfrac{1}{60}\right)^2 a_{\overline{59|}} = 524 \end{aligned}$$

Answer is C.

Question 10

The outstanding balance of the annuity at any point in time is equal to the present value of the future payments.

After 9 years, there remain payments of .5X for 11 years plus an additional payment of .5X in year 10. After 10 years, all remaining payments are .5X.

Outstanding balance after 9 years $= .5X\,a_{\overline{11|}} + .5Xv = 4.6294X$

Outstanding balance after 10 years $= .5X\,a_{\overline{10|}} = 3.8609X$

Outstanding balance after 11 years $= .5X\,a_{\overline{9|}} = 3.5539X$

The principal repaid in a particular payment is the difference between the outstanding balance at the beginning of the year and the outstanding balance at the end of the year.

$$Y = \frac{4.6294X - 3.8609X}{3.8609X - 3.5539X} = 2.5033$$

Answer is C.

Question 11

Contribution in 2005: 6% of \$50,000 = \$3,000
Each subsequent contribution will increase by 4%, as the salary increases.

$$\begin{aligned}X &= 3{,}000(1.05^{24}) + 3{,}000(1.04)(1.05^{23}) + \ldots + 3{,}000(1.04^{24})\\
&= 3{,}000(1.04^{24}) \times [(\tfrac{1.05}{1.04})^{24} + (\tfrac{1.05}{1.04})^{23} + \ldots + (\tfrac{1.05}{1.04}) + 1]\\
&= 3{,}000(1.04^{24}) \times [(1.0096)^{24} + (1.0096)^{23} + \ldots + 1.0096 + 1]\\
&= 3{,}000(1.04^{24}) \times s_{\overline{25}|.0096}\\
&= 216{,}114\end{aligned}$$

Answer is D.

Question 12

$l_{64}^{(T)} = l_{63}^{(T)} \times (1 - q_{63}^{(1)} - q_{63}^{(2)}) \Rightarrow l_{64}^{(T)} = 500 \times (1 - .05 - .50) \Rightarrow l_{64}^{(T)} = 225$

$p_{63}^{(T)} = 1 - q_{63}^{(1)} - q_{63}^{(2)} = 1 - .05 - .50 = .45$

${}_2q_{63}^{(2)} = q_{63}^{(2)} + p_{63}^{(T)}\, q_{64}^{(2)} \Rightarrow .60 = .50 + (.45 \times q_{64}^{(2)}) \Rightarrow q_{64}^{(2)} = .2222$

${}_{1|}q_{63}^{(1)} = p_{63}^{(T)}\, q_{64}^{(1)} \Rightarrow .07 = .45 \times q_{64}^{(1)} \Rightarrow q_{64}^{(1)} = .1556$

$l_{65}^{(T)} = l_{64}^{(T)} \times (1 - q_{64}^{(1)} - q_{64}^{(2)}) \Rightarrow l_{65}^{(T)} = 225 \times (1 - .1556 - .2222)$
$\Rightarrow l_{65}^{(T)} = 140$

$p_{64}^{(T)} = 1 - q_{64}^{(1)} - q_{64}^{(2)} = 1 - .1556 - .2222 = .6222$

${}_{2|}q_{63}^{(1)} = {}_2p_{63}^{(T)}\, q_{65}^{(1)} \Rightarrow .042 = .45 \times .6222 \times q_{65}^{(1)} \Rightarrow q_{65}^{(1)} = .15$

$d_{65}^{(1)} = l_{65}^{(T)}\, q_{65}^{(1)} = 140 \times .15 = 21$

$l_{65}^{(T)} - l_{66}^{(T)} = d_{65}^{(1)} + d_{65}^{(2)} \Rightarrow 140 - 0 = 21 + d_{65}^{(2)} \Rightarrow d_{65}^{(2)} = 119$

Answer is E.

Question 13

There is a constant force of mortality since μ_x is a constant. From formula 1.12 in the Jordan text ("Life Contingencies"),

$p_x = e^{-.1}$ and $_np_x = e^{-.1n}$

The probability of a life surviving 10 years is:

$_{10}p_x = e^{-1}$

For two independent lives age 30 and 50 to die within 10 years of each other, the one who dies last cannot live for more than 10 years after the other dies. This has a probability of:

$1 - {}_{10}p_x = 1 - e^{-1}$

Answer is E.

Question 14

From an approximation on page 174 of the Jordan text ("Life Contingencies"),

$$
\begin{aligned}
X = \mathring{e}_{60:\overline{1.5}|} &= e_{60:\overline{1.5}|} + ½(1 - {}_{1.5}p_{60}) \\
&= [p_{60} + (p_{60})(½p_{61})] + ½[1 - (p_{60})(1 - ½q_{61})] \\
&= [.98 + ½(.98)(.978)] + ½[1 - (.98)(1 - ½(.022))] \\
&= 1.4746
\end{aligned}
$$

Answer is D.

Question 15

The following relationships from formula 3.11 of the Jordan text ("Life Contingencies") can be used to determine the life annuity values:

$A_x = 1 - d\ddot{a}_x$ and $A_{x:\overline{n}|} = 1 - d\ddot{a}_{x:\overline{n}|}$

From formula 3.7 of the Jordan text ("Life Contingencies"): $A_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + {}_nE_x$

Also recall that $d = i/(1 + i)$

Finally, the present value of a temporary annuity is equal to the present value of the life annuity reduced by the present value of the annuity payments after the temporary period:

$\ddot{a}_{x:\overline{n}|} = \ddot{a}_x - {}_nE_x\, \ddot{a}_{x+n}$

Using the given data,

$A_{40} = 1 - d\ddot{a}_{40} \quad \Rightarrow \quad 0.30 = 1 - \frac{.04}{1.04}\ddot{a}_{40} \quad \Rightarrow \quad \ddot{a}_{40} = 18.2$

$A_{50} = 1 - d\ddot{a}_{50} \quad \Rightarrow \quad 0.35 = 1 - \frac{.04}{1.04}\ddot{a}_{50} \quad \Rightarrow \quad \ddot{a}_{50} = 16.9$

$A_{40:\overline{10}|} = 1 - d\ddot{a}_{40:\overline{10}|}$

$\ddot{a}_{40:\overline{10}|} = (1 - A_{40:\overline{10}|})/d$

$\ddot{a}_{40:\overline{10}|} = (1 - A^{1}_{40:\overline{10}|} - {}_{10}E_{40})/d = (1 - 0.09 - {}_{10}E_{40})/d = (0.91 - {}_{10}E_{40})/d$

Also, $\ddot{a}_{40:\overline{10}|} = \ddot{a}_{40} - {}_{10}E_{40}\,\ddot{a}_{50} = 18.2 - 16.9\,{}_{10}E_{40}$

So, $18.2 - 16.9\,{}_{10}E_{40} = (0.91 - {}_{10}E_{40})/d \quad \Rightarrow \quad {}_{10}E_{40} = 0.6$

Setting the actuarially equivalent payment options equal,

$10{,}000 = X(\ddot{a}_{\overline{10}|} + {}_{10|}\ddot{a}_{40}) = X(\ddot{a}_{\overline{10}|} + {}_{10}E_{40}\,\ddot{a}_{50}) = X(8.4353 + (0.6 \times 16.9))$

$X = 538.35$

Answer is A.

Question 16

Annual retirement benefit at each possible retirement age for the employee:

Age 63: \$15 × 23 years of service × 12 = \$4,140
Age 64: \$15 × 24 years of service × 12 = \$4,320
Age 65: \$15 × 25 years of service × 12 = \$4,500

The present value of the benefits for each possible retirement age at age 45 is:

Age 63: $\$4{,}140 \times \ddot{a}_{63}^{(12)} \times v^{18} = \$4{,}140 \times 10.0 \times .4155 = \$17{,}202$

Age 64: $\$4{,}320 \times \ddot{a}_{64}^{(12)} \times v^{19} = \$4{,}320 \times 9.5 \times .3957 = \$16{,}240$

Age 65: $\$4{,}500 \times \ddot{a}_{65}^{(12)} \times v^{20} = \$4{,}500 \times 9.0 \times .3769 = \$15{,}264$

The probability of retirement at each age is:

Age 63: 0.2
Age 64: 0.8 × 0.3 = 0.24
Age 65: 0.8 × 0.7 × 1.0 = 0.56

The present value of the benefits at each age must be multiplied by the probability of retirement at that age:

X = (\$17,202 × 0.2) + (\$16,240 × 0.24) + (\$15,264 × 0.56) = \$15,886

Answer is D.

Question 17

$i^{(12)}/12 = .06/12 = .005$

The present value of the first year's payments at the beginning of the first year is:

$2000\, a_{\overline{12}|.005} = 23{,}238$

The annual effective rate of interest is: $i = (1.005)^{12} - 1 = .0617$

The present value can be written as:

$X = 23{,}238 \times (1 + 1.05v + (1.05v)^2 + \ldots + (1.05v)^{19})$

Note that $1.05v$ can be evaluated as:

$1.05v = 1.05/1.0617 = 1/(1.0617/1.05) = 1/1.0111$

Therefore, the combination of the interest rate and the salary increases results in an implicit interest rate of 1.11%. The present value is:

$X = 23{,}238\, \ddot{a}_{\overline{20}|.0111} = 419{,}338$

Answer is D.

Question 18

Annual interest received from fund A per \$150 invested = \$150 × .15 = \$22.50

This is invested at the end of each year into fund B. The initial investment into fund B will be at the end of the second year since the first investment into fund A does not occur until the end of the first year. Note that the amount invested into fund B each year increases incrementally since an additional \$150 of principal is invested each year into fund A. There will only be 19 payments into fund B (not 20) since there is no payment in the first year.

$$
\begin{aligned}
X &= (\$22.50 \times 1.11^{18}) + (2 \times \$22.50 \times 1.11^{17}) + \ldots + (19 \times \$22.50) \\
&= \$22.50\,(Is)_{\overline{19|}.11} \\
&= \$22.50 \times 1.11^{19} \times (Ia)_{\overline{19|}.11} \\
&= \$22.50 \times 1.11^{19} \times \frac{\ddot{a}_{\overline{19|}} - 19v^{19}}{.11} \\
&= \$9{,}041
\end{aligned}
$$

Answer is A.

Question 19

Duration is the weighted average of the present value of future payments, with the weighting based upon the time the payment is made. Recall the formula for duration (Macaulay duration):

$$\bar{d} = \frac{\sum_{t=1}^{n} t v^t R_t}{\sum_{t=1}^{n} v^t R_t}$$, where R_t represents the payment at time t.

$$X = \frac{10{,}000v + 3 \times (20{,}000v^3) + 4 \times (15{,}000v^4)}{10{,}000v + 20{,}000v^3 + 15{,}000v^4} = 2.8167$$

Answer is D.

Question 20

$e_x = \overset{\circ}{e}_x - ½ = 2 - ½ = 1.5$

$e_{x+1} = \overset{\circ}{e}_{x+1} - ½ = 1.5 - ½ = 1$

$e_x = p_x + {}_2p_x + {}_3p_x + \ldots = p_x(1 + e_{x+1})$

Substituting,

$1.5 = p_x(1 + 1) \quad \Rightarrow \quad p_x = .75$

The probability that neither Smith nor Brown dies in 2005 is: $p_x \times p_{x+1}$

The probability that Brown dies during 2005 is: $1 - p_{x+1}$

Therefore,

$p_x \times p_{x+1} = 1.5 \times (1 - p_{x+1}) = 1.5 – 1.5\ p_{x+1}$
$.75p_{x+1} = 1.5 – 1.5\ p_{x+1}$
$p_{x+1} = ⅔$

$e_{x+1} = p_{x+1} + {}_2p_{x+1} + {}_3p_{x+1} + \ldots = p_{x+1}(1 + e_{x+2})$

Substituting,

$1 = ⅔(1 + e_{x+2})$
$e_{x+2} = .5$

Answer is C.

Question 21

The purchase price of a bond is equal to the present value of the redemption amount plus the present value of the future coupon payments. The semi-annual coupon payments of this bond are \$25 (½ × 5% × \$1,000).

Converting the yield rates to semi-annual effective rates:

$i' = 1.04^{½} - 1 = .0198$
$j' = 1.06^{½} - 1 = .0296$

The purchase price at a 4% yield is:

$$1{,}000\, v_{4\%}^{10} + 25\, a_{\overline{20}|.0198} = 676 + 410 = 1{,}086$$

The premium is the excess of the price of the bond over the redemption amount:

$X = 1{,}086 - 1{,}000 = 86$

The purchase price at a 6% yield is:

$$1{,}000\, v_{6\%}^{10} + 25\, a_{\overline{20}|.0296} = 558 + 373 = 931$$

The discount is the excess of the redemption amount of the bond over the purchase price:

$Y = 1{,}000 - 931 = 69$

$X - Y = 86 - 69 = 17$

Answer is E.

Question 22

I. As interest rates change, the changes in the present value of the future bond payments (the coupons and the redemption amount) change more drastically the longer the time frame involved. This statement is true.

II. The amount of the coupon is a fixed future payment that is dependent only on the bond's coupon rate. This coupon amount (like the redemption amount) has no bearing on the bond's sensitivity to interest rate fluctuations. This statement is false.

III. A bond that is callable sooner rather than later would be less likely to be influenced by a change in interest rates, since the time frame involved in calculating the present value is shorter rather than longer. This statement is false.

Answer is E.

Question 23

${}_{15|15}q_{25} = {}_{15}p_{25} \times {}_{15}q_{40} = 0.8108 \times {}_{15}q_{40} = 0.2027$

${}_{15}q_{40} = 0.25$

${}_{15}p_{40} = 1 - {}_{15}q_{40} = 0.75$

${}_{30|10}q_{25} = {}_{15}p_{25} \times {}_{15}p_{25} \times {}_{10}q_{55} = 0.8108 \times 0.75 \times 0.2222 = 0.1351$

Answer is D.

Question 24

$_{5|}\ddot{a}_{55} = \ddot{a}_{55} - \ddot{a}_{55:\overline{5}|}$ and $_{5|}\ddot{a}_{55} = {}_5E_{55}\,\ddot{a}_{60}$

$\ddot{a}_{55} - \ddot{a}_{55:\overline{5}|} = {}_5E_{55}\,\ddot{a}_{60} \quad \Rightarrow \quad 11.2751 - 4.3122 = 10.2758\,{}_5E_{55}$

$\Rightarrow \quad {}_5E_{55} = 0.6776$

$_{5|}\ddot{a}_{60} = \ddot{a}_{60} - \ddot{a}_{60:\overline{5}|}$ and $_{5|}\ddot{a}_{60} = {}_5E_{60}\,\ddot{a}_{65}$

$\ddot{a}_{60} - \ddot{a}_{60:\overline{5}|} = {}_5E_{60}\,\ddot{a}_{65} \quad \Rightarrow \quad 10.2758 - 4.2707 = 9.1301\,{}_5E_{60}$

$\Rightarrow \quad {}_5E_{60} = 0.6577$

$_{10|}\ddot{a}_{55} = {}_5E_{55} \times {}_5E_{60}\,\ddot{a}_{65} = 0.6776 \times 0.6577 \times 9.1301 = 4.0689$

$\ddot{a}_{55:\overline{10}|} = \ddot{a}_{55} - {}_{10|}\ddot{a}_{55} = 11.2751 - 4.0689 = 7.2062$

Setting the two actuarially equivalent annuities equal,

$X(\ddot{a}_{\overline{10}|} + {}_{10|}\ddot{a}_{55}) = 5{,}000\,\ddot{a}_{55} + 2{,}500\,\ddot{a}_{55:\overline{10}|} + 2{,}500\,\ddot{a}_{55:\overline{5}|}$

$X(7.5152 + 4.0689) = 85{,}172$

$X = 7{,}352$

Answer is A.

Question 25

The force of interest (δ) during the first year is 0.07 (0.06 + 0.01), and during the second year it is 0.08 (0.06 + (2 × 0.01)).

The force of interest can be converted to an annual effective rate of interest by thinking of the force of interest as a nominal annual rate of interest compounded very often (say 100,000 times per year). So, think of δ as being $i^{(100,000)}$.

For $\delta_1 = .07$, $i_1 = (1 + \frac{.07}{100{,}000})^{100{,}000} - 1 = .0725$

For $\delta_2 = .08$, $i_2 = (1 + \frac{.08}{100{,}000})^{100{,}000} - 1 = .0833$

The present value of the payments is equal to the loan amount.

$10{,}000 = X/1.0725 + 1.1X/(1.0725 \times 1.0833) \quad \Rightarrow \quad X = 5{,}321$

Answer is C.

Question 26

The term cost is equal to the present value of the disability benefit if disability occurs during 2005. Disability benefits are only paid if recovery does not occur, and if the disabled life is still alive. The probability of a disabled life **not** recovering in any year is 97% (100% - 3%), and the probability of a disabled life **not** dying in any year is 92% (100% - 8%).

$$
\begin{aligned}
Y &= q_{40}^{(i)} \times \$25{,}000 \times (v + p_{40}^{\prime(rec)}\, p_{40}^{\prime(d)}\, v^2 + \ldots + {}_{24}p_{40}^{\prime(rec)}\; {}_{24}p_{40}^{\prime(d)}\, v^{25}) \\
&= 0.005 \times \$25{,}000 \times (v + (0.97 \times 0.92 \times v^2) + \ldots + (0.97 \times 0.92)^{24} \times v^{25})) \\
&= 0.005 \times \$25{,}000 \div (0.97 \times 0.92) \\
&\qquad \times ((0.97 \times 0.92 \times v) + (0.97 \times 0.92 \times v)^2 + \ldots + (0.97 \times 0.92 \times v)^{25})
\end{aligned}
$$

Note that $0.97 \times 0.92 \times v$ can be evaluated as:

$$0.97 \times 0.92 \times v = 0.97 \times 0.92 / 1.07 = 1/(1.07/(0.97 \times 0.92)) = 1/1.1990$$

The combination of the interest rate and the probabilities results in an implicit interest rate of 19.90%.

$$Y = 0.005 \times \$25{,}000 \div (0.97 \times 0.92) \times a_{\overline{25}|.1990} = \$696.34$$

Answer is A.

Question 27

The termination rate used for all participants with 2 or more years of service is the regular (non-select) rate since the select period is two years. The expected number of terminations during 2005 of the participants with 2 or more years of service is:

$$900 \times q_{40}^{(w)} = 900 \times 0.10 = 90$$

Of the remaining 810 participants as of 1/1/2006, the expected number of terminations during 2006 is:

$$810 \times q_{41}^{(w)} = 810 \times (0.10 - 0.003) = 78.57$$

For the participants covered under the select table, there are 200 who are age 40. The expected number of terminations from this group during 2006 is:

$$200 \times (1 - q_{[40]}^{(w)}) \times q_{[40]+1}^{(w)} = 200 \times (1 - q_{[40]}^{(w)}) \times (q_{[40]}^{(w)} - 0.02) = 204\,q_{[40]}^{(w)} - 4 - 200(q_{[40]}^{(w)})^2$$

There are also 150 participants who are age 41 with one year of service. The expected number of terminations from this group during 2006 is:

$$\begin{aligned} 150 \times (1 - q_{[40]+1}^{(w)}) \times q_{42}^{(w)} &= 150 \times (1 - (q_{[40]}^{(w)} - 0.02)) \times (0.10 - (0.003 \times 2)) \\ &= 14.1 \times (1.02 - q_{[40]}^{(w)}) \\ &= 14.382 - 14.1\,q_{[40]}^{(w)} \end{aligned}$$

Since there are a total of 120 expected to terminate in 2006,

$$120 = 78.57 + [204\,q_{[40]}^{(w)} - 4 - 200(q_{[40]}^{(w)})^2] + [14.382 - 14.1\,q_{[40]}^{(w)}]$$

$$- 200(q_{[40]}^{(w)})^2 + 189.9\,q_{[40]}^{(w)} - 31.048 = 0$$

Recall the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, a = -200, b = 189.9, and c = -31.048.

Substituting,

$$q_{[40]}^{(w)} = \frac{-189.9 \pm \sqrt{(189.9)^2 - (4)(-200)(-31.048)}}{(2)(-200)} = 0.2099 \text{ or } 0.7396$$

Clearly, only $q_{[40]}^{(w)} = 0.2099$ is a reasonable answer.

Answer is E.

Question 28

$$p_{40}^{(T)} = \frac{l_{41}^{(T)}}{l_{40}^{(T)}} = \frac{417{,}362}{500{,}000} = 0.834724$$

Recall that $p_{40}^{(T)} = p_{40}'^{(1)} \times p_{40}'^{(2)}$.

$0.834724 = (1 - q_{40}'^{(1)}) \times p_{40}'^{(2)} = 0.9066\, p_{40}'^{(2)} \quad \Rightarrow \quad p_{40}'^{(2)} = 0.9207$

Recall the standard approximation $q_x^{(1)} = q_x'^{(1)} \times (1 - ½\, q_x'^{(2)})$.

$q_{40}^{(1)} = 0.0934 \times (1 - (½ \times 0.0793)) = 0.0897$
$q_{40}^{(2)} = 0.0793 \times (1 - (½ \times 0.0934)) = 0.0756$

$$X = m_{40}^{(2)} = \frac{q_{40}^{(2)}}{1 - \frac{1}{2}(q_{40}^{(1)} + q_{40}^{(2)})} = \frac{0.0756}{1 - \frac{1}{2}(0.0897 + 0.0756)} = 0.0824$$

Answer is D.

Question 29

Recall from first principles that:

$\ddot{a}_{64} = 1 + vp_{64} + v^2{}_2p_{64} + v^3{}_3p_{64} + \ldots = 1 + vp_{64}(1 + vp_{65} + v^2{}_2p_{65} + \ldots) = 1 + vp_{64}\,\ddot{a}_{65}$

Therefore, $\ddot{a}_{64} = 1 + [(1/1.07) \times 0.991315 \times 10.8207] = 11.0250$

$$\begin{aligned}100{,}000 &= X\ddot{a}_{64:65} + (X - 3{,}000)\,\ddot{a}_{64|65} + (X - 3{,}000)\,\ddot{a}_{65|64} \\ &= X\ddot{a}_{64:65} + (X - 3{,}000)(\ddot{a}_{65} - \ddot{a}_{64:65}) + (X - 3{,}000)(\ddot{a}_{64} - \ddot{a}_{64:65}) \\ &= 9.1707X + (X - 3{,}000)(10.8207 - 9.1707) + (X - 3{,}000)(11.0250 - 9.1707) \\ &= 12.675X - 10{,}513\end{aligned}$$

X = 8,719

Answer is C.