SOCIETY OF ACTUARIES

AMERICAN SOCIETY OF PENSION ACTUARIES

JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

**MAY 1999 COURSE 141 (EA1) SEGMENT A**

**JOINT BOARD BASIC EXAMINATION**

This is the May 1999 examination which has been released to
the public by the administering organizations.

Spring 1999
EA -1A

## Conditions Generally Applicable to
## All EA-1 Segment A Examination Questions

The following conditions should be considered a part of the data for each question, unless otherwise stated or implied.

(1)   The normal retirement age is 65.

(2)   Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.

(3)   There are no preretirement death or disability benefits.

(4)   Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.

(5)   Interest rates which are compounded more frequently than annually are expressed as nominal rates.

(6)   Where multiple lives are involved, future lifetimes are assumed to be independent.

(7)   The term "gross single premium" is equivalent to "contract single premium"; the term "net single premium" is equivalent to "benefit single premium".

(8)   There are no policy loans in effect.

(9)   For a bond, the face amount and the redemption value are the same.

(10)   Interest rate equals yield rate.

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## Data for Question 1

Market value of a pension fund:

| Date | Value |
| --- | --- |
| 1/1/99 | $100,000 |
| 4/1/99 | 90,000 |
| 7/1/99 | 95,000 |
| 10/1/99 | 185,000 |
| 1/1/2000 | 180,000 |

Contributions and Benefit Payments:

| Date | Contributions | Benefit Payments |
| --- | --- | --- |
| 3/31/99 | $0 | $20,000 |
| 9/30/99 | 75,000 | 0 |

Rates of return on fund:

Time weighted = t
Dollar weighted = d
Dollar weighted assuming cash flows at mid-year = m

## Question 1

Which of the following is true?

[A]     $t > m > d$

[B]     $t > d > m$

[C]     $m > t > d$

[D]     $d > t > m$

[E]     None of the above

Data for Question 2

Terms of a 25-year annuity certain:

Date of first payment:  12/31/99.

Frequency of payments:  Annual.

Amount of each payment:

First 10 years:  $1,000 per year.
Next 10 years:  $1,500 per year.
Final 5 years:   $2,000 per year.

Interest rate:  8% per year, compounded semiannually.

Question 2

In what range is the present value of the annuity as of 1/1/99?

[A]    Less than $12,500

[B]    $12,500 but less than $13,000

[C]    $13,000 but less than $13,500

[D]    $13,500 but less than $14,000

[E]    $14,000 or more

## Data for Question 3

Terms of two actuarially equivalent annuities:

Annuity A: $500 at the end of each of the first 3 months, and $1,000 at the end of each of the next 9 months.

Annuity B: $P at the end of each of the first 2 quarters, and $2P at the end of each of the next 2 quarters.

Interest rate: 8% per year, compounded monthly.

## Question 3

In what range is $P?

[A] Less than $1,770

[B] $1,770 but less than $1,800

[C] $1,800 but less than $1,830

[D] $1,830 but less than $1,860

[E] $1,860 or more

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Data for Question 4

Date of a loan:  1/1/99.

Interest rate:  10.8% per year, compounded monthly.

Date of first repayment:  2/1/99.

Frequency of repayments:  Monthly.

Number of repayments:  360.


Question 4

In what range is the modified duration of the loan?

[A]     Less than 95 months

[B]     95 months but less than 96 months

[C]     96 months but less than 97 months

[D]     97 months but less than 98 months

[E]     98 months or more

## Data for Question 5

Market value of a pension fund on 12/31/98: $1,750,000.

Benefit payments are made on the first day of each calendar quarter.

Contributions are made on the last day of each calendar quarter and included in the ending balance.

Time-weighted rate of return for 1999: 8.79%.

1999 fund data:

| Quarter | Benefit Payments | Contributions | Quarterly Return | Ending Balance |
|---------|------------------|---------------|------------------|----------------|
| 1 | $ 22,000 | $30,000 | 1.02% | $1,775,626 |
| 2 | 43,000 | 85,000 | 5.19% | 1,907,549 |
| 3 | 27,994 | y | -2.26% | 1,937,033 |
| 4 | 39,228 | 27,617 | j% | z |

## Question 5

In what range is the dollar-weighted rate of return for 1999?

[A]   Less than 8.68%

[B]   8.68% but less than 8.73%

[C]   8.73% but less than 8.78%

[D]   8.78% but less than 8.83%

[E]   8.83% or more

Data for Question 6

Date of a loan:  1/1/99.

Amount of loan:  $10,000.

Date of first repayment:  2/1/99.

Frequency of repayments:  Monthly.

Term of loan:  10 years.

Interest rate:  7% per year, compounded annually.

Prepayment penalty:  10% of outstanding loan amount.

The borrower establishes a savings account to which annual payments of $P are made. The savings account earns 9% per year, compounded monthly. The first payment to the savings account is made on 12/31/99. Just prior to the 53$^{rd}$ scheduled loan repayment the savings account balance is sufficient to pay both the outstanding loan amount and the prepayment penalty.

Question 6

In what range is $P?

[A]    Less than $1,475

[B]    $1,475 but less than $1,485

[C]    $1,485 but less than $1,495

[D]    $1,495 but less than $1,505

[E]    $1,505 or more

Data for Question 7

Smith takes out a mortgage on 1/1/94 with the following provisions:

Amount of mortgage: $100,000.
Interest rate: 12% per year, compounded monthly.
Number of repayments: 360.
Frequency of repayments: Monthly.
Date of first repayment: 1/31/94.

On 12/31/98 , after making the 12/31/98 repayment, Smith repays an additional $3,000 to reduce the outstanding balance of his mortgage. He will continue to make the same monthly repayments he has been making (or the outstanding balance of the mortgage, if less) until the outstanding balance of his mortgage is reduced to zero.

Question 7

In what range is the total interest paid by Smith?

[A]    Less than $225,000

[B]    $225,000 but less than $235,000

[C]    $235,000 but less than $245,000

[D]    $245,000 but less than $255,000

[E]    $255,000 or more

## Data for Question 8

Terms of a perpetuity:

Effective date: 1/1/99.
Frequency of payments: Annual.
Date of first payment: 12/31/99.
Amount of each payment: $2X.
Present value of future payments as of 1/1/2009: $P.

Terms of a 10-year annuity certain:

Effective date: 1/1/99.
Frequency of payments: Annual.
Date of first payment: 12/31/99.
Amount of each payment: $X.
Accumulated value of payments as of 1/1/2009: $P/2.

Interest rate: i%.

## Question 8

In what range is i%?

[A]    Less than 7.25%

[B]    7.25% but less than 7.75%

[C]    7.75% but less than 8.25%

[D]    8.25% but less than 8.75%

[E]    8.75% or more

## Data for Question 9

Fund balance as of 1/1/99:  $12,000.

Deposits to the fund:  $100 on the last day of each month for 5 years.

First deposit:  1/31/99.

Withdrawals from the fund:  $1,000 on the first day of each quarter.

First withdrawal:  1/1/2006.

No other deposits or withdrawals are made.

Interest rate:  8% per year, compounded monthly.


## Question 9

In what range is the fund balance as of 12/31/2010?

[A]  Less than $13,500

[B]  $13,500 but less than $15,000

[C]  $15,000 but less than $16,500

[D]  $16,500 but less than $18,000

[E]  $18,000 or more

Data for Question 10

Purchase date of a perpetuity: 1/1/99.

Date of first payment: 3/31/99.

Frequency of payments: Quarterly.

Quarterly payments during each year as follows:

| Quarter | Amount |
|---------|--------|
| 1 | $100 |
| 2 | 200 |
| 3 | 300 |
| 4 | 400 |

Interest rate: 10% per year, compounded annually.

Question 10

In what range is the purchase price of the perpetuity?

[A]    Less than $10,125

[B]    $10,125 but less than $10,250

[C]    $10,250 but less than $10,375

[D]    $10,375 but less than $10,500

[E]    $10,500 or more

Data for Question 11

Face amount of a bond: $100,000.

Purchase date: 1/1/99.

Coupon rate: 6% per year, payable semiannually on each 6/30 and 12/31.

Redemption date: 1/1/2009.

Redemption amount: $100,000.

Coupons are reinvested at 5% per year, compounded semiannually.

Overall yield rate to purchaser: 5.5% per year, compounded semiannually.

Question 11

In what range is the purchase price of the bond?

[A]   Less than $99,000

[B]   $99,000 but less than $100,500

[C]   $100,500 but less than $102,000

[D]   $102,000 but less than $103,500

[E]   $103,500 or more

Data for Question 12

Type of insurance policy: Whole life.

Age of insured at issue: 30.

Death benefit: Payable at the end of the year of death.

Net annual premium (Annual benefit premium): $45 per $1,000 of face amount, payable for life.

Net single premium (Single benefit premium): $Y per $1,000 of face amount.

Interest rate: 6% per year, compounded annually.

Question 12

In what range is $Y?

[A]    Less than $400

[B]    $400 but less than $450

[C]    $450 but less than $500

[D]    $500 but less than $550

[E]    $550 or more

## Data for Question 13

Value of an inheritance: $140,000.

Number of beneficiaries: 3.

Age of each beneficiary: y.

Terms of a monthly annuity, payable at the beginning of each month, to be purchased with inheritance:

- $400 to each beneficiary while all are alive.
- $450 to each surviving beneficiary after the first death.
- $X to the surviving beneficiary after the second death.

Selected values:

$$\ddot{a}^{(12)}_{y} = 9.194 \qquad \ddot{a}^{(12)}_{yy} = 7.354 \qquad \ddot{a}^{(12)}_{yyy} = 6.258$$

## Question 13

In what range is the value of $X?

[A]    Less than $200

[B]    $200 but less than $400

[C]    $400 but less than $600

[D]    $600 but less than $800

[E]    $800 or more

## Data for Question 14

Under a two decrement table used for a pension valuation the absolute rate of death is 2% and the absolute rate of termination is 4% for all ages.

The pension plan provides for a death benefit after 5 years of service and for a retirement benefit for anyone who terminates after attainment of age 55 with 5 years of service.

An employee was hired at age 58 and is currently age 60.

## Question 14

In what range is the probability that the employee receives either a death or retirement benefit from the plan?

[A]   Less than 0.828

[B]   0.828 but less than 0.830

[C]   0.830 but less than 0.832

[D]   0.832 but less than 0.834

[E]   0.834 or more

Data for Question 15

Ages as of 1/1/99:

> Smith: 60.
> Green: 63.
> Brown: 65.

Selected values from a mortality table:

$$l_{60} = 1,000 \qquad l_{60+t} = 1,000 - [19t - 1], \text{ where } t \geq 1$$

Question 15

In what range is the probability that Green will not die in 2004 and at least one of Smith and Brown will die in 2004?

[A]   Less than .0365

[B]   .0365 but less than .0375

[C]   .0375 but less than .0385

[D]   .0385 but less than .0395

[E]   .0395 or more

Data for Question 16

Selected annuity values:

| x | $\ddot{a}_x$ |
|---|---|
| 50 | 12.07872 |
| 51 | 11.92117 |
| 52 | 11.75854 |

Interest rate:  7% per year, compounded annually.

Question 16

In what range is the probability an annuitant age 50 would die before age 52?

[A]    Less than .005

[B]    .005 but less than .010

[C]    .010 but less than .015

[D]    .015 but less than .020

[E]    .020 or more

Data for Question 17

Terms of three annuity contracts:

|  | Contract 1 | Contract 2 | Contract 3 |
|---|---|---|---|
| Age at issue | 30 | 30 | 30 |
| Net single premium | $6,000 | $75,000 | $P |
| Age at first annuity payment | 31 | 30 | 31 |
| Maximum number of payments | 15 | 16 | 15 |
| Frequency of annuity payments | Annual | Annual | Annual |
| Amount of first annuity payment | $1,000 | $5,000 | $7,500 |
| Amount of increase (decrease) in each subsequent payment | $0 | $1,000 | ($500) |

The net single premium under each contract is based on the same mortality and interest assumptions.

Question 17

In what range is $P?

[A]    Less than $23,000

[B]    $23,000 but less than $25,000

[C]    $25,000 but less than $27,000

[D]    $27,000 but less than $29,000

[E]    $29,000 or more

## Data for Question 18

Data for pension plan participant Smith:

Date of birth: 1/1/54.
Date of termination: 1/1/99.
Normal retirement benefit: $7,200 commencing at age 65, payable annually for life.
Benefit Option A: $X commencing at age 55, payable annually for life with 15 years certain.

Option A is actuarially equivalent to the normal retirement benefit.

Selected values:

$$\ddot{a}_{60} = 10.2733 \qquad {}_{10}E_{45} = .4808$$
$$\ddot{a}_{65} = 9.1941 \qquad {}_{10}E_{55} = .4413$$
$$\ddot{a}_{70} = 8.0605 \qquad {}_{20}E_{45} = .2122$$
$$\ddot{a}_{75} = 6.8749 \qquad {}_{25}E_{45} = .1317$$

Interest rate: 7% per year, compounded annually.

## Question 18

In what range is $X?

[A]  Less than $2,200

[B]  $2,200 but less than $2,300

[C]  $2,300 but less than $2,400

[D]  $2,400 but less than $2,500

[E]  $2,500 or more

Data for Question 19

Terms of a life annuity due:

> Date of first payment: 1/1/99.
> Age at first payment: 60.
> Amount of each payment: $15,000.
> Frequency of payments: Annual.
> Interest rate: 7% per year, compounded annually.

The premium for the annuity is based upon mortality rates based on a standard table with an improved rate of survivorship at age 60 equal to $p_{60}+1\%$, where $p_{60}$ is from the standard table.

Selected values from the standard table:

$$1000 \, A_{60:\,\overline{1}|}^{\,1} = 921$$

$$1000 \, A_{60} = 328$$

Question 19

In what range is the net single premium for the annuity as of 1/1/99?

[A]   Less than $139,000

[B]   $139,000 but less than $147,000

[C]   $147,000 but less than $155,000

[D]   $155,000 but less than $163,000

[E]   $163,000 or more

Data for Question 20

Selected values from a mortality table with a one-year select period:

$q_{[x]} = 0.6 q_x$

$a_{40} = 16$

The force of mortality, $\mu$, during the select period is ln (1.04) for all ages.

Question 20

In what range is $a_{[40]}$?

[A]    Less than 16.15

[B]    16.15 but less than 16.30

[C]    16.30 but less than 16.45

[D]    16.45 but less than 16.60

[E]    16.60 or more

## Data for Question 21

Selected values:

$$_np_{xx} = .25$$

$$p_{x+n} = .5$$

## Question 21

In what range is $_nq_x + {}_nq_{xx} - {}_{n|}q_{xxx}$?

[A]  Less than 1.14

[B]  1.14 but less than 1.16

[C]  1.16 but less than 1.18

[D]  1.18 but less than 1.20

[E]  1.20 or more

## Data for Question 22

Effective date of a whole life policy: 1/1/99.

Death benefit: $100,000 payable at end of the year of death.

Purchase age: 45.

Net annual premium (Annual benefit premium): $1,548 payable 1/1.

Selected values:

| x | $N_x$ |
|---|---|
| 45 | 4,450 |
| 46 | 4,208 |
| 65 | 1,060 |
| 66 | 960 |

## Question 22

In what range is $100,000 $_{20}V_{45}$?

[A]   Less than $40,000

[B]   $40,000 but less than $41,000

[C]   $41,000 but less than $42,000

[D]   $42,000 but less than $43,000

[E]   $43,000 or more

## Data for Question 23

$l_x = 110 - x$.

Interest rate: 7% per year, compounded annually.

## Question 23

In what range is $\ddot{a}_{65}$?

[A]  Less than 10.60

[B]  10.60 but less than 11.60

[C]  11.60 but less than 12.60

[D]  12.60 but less than 13.60

[E]  13.60 or more

## Data for Question 24

Data relative to a 4-year temporary life annuity:

Purchase date: 1/1/99.

Age at purchase: 65.

Present value as of 1/1/99: $100,000.

Date of first payment: 12/31/99.

Frequency of payments: Annual.

Interest rate: 7% per year, compounded annually.

Selected values from a 3-year select mortality table:

| [x] | $l_{[x]}$ | $l_{[x]+1}$ | $l_{[x]+2}$ | $l_{x+3}$ | x+3 |
|-----|-----------|-------------|-------------|-----------|-----|
| 62 | 10,000,000 | 9,844,074 | 9,662,671 | 9,460,642 | 65 |
| 63 | 9,831,050 | 9,662,801 | 9,465,583 | 9,247,191 | 66 |
| 64 | 9,642,924 | 9,463,045 | 9,251,733 | 9,017,426 | 67 |
| 65 | 9,435,643 | 9,245,121 | 9,020,841 | 8,771,863 | 68 |
| 66 | 9,206,484 | 9,007,739 | 8,773,003 | 8,511,918 | 69 |
| 67 | 8,963,417 | 8,757,765 | 8,512,335 | 8,238,915 | 70 |
| 68 | 8,703,733 | 8,492,221 | 8,237,205 | 7,952,671 | 71 |
| 69 | 8,414,800 | 7,922,545 | 7,648,142 | 7,340,327 | 72 |

## Question 24

In what range is the annual payment?

[A]   Less than $31,200

[B]   $31,200 but less than $31,300

[C]   $31,300 but less than $31,400

[D]   $31,400 but less than $31,500

[E]   $31,500 or more

Data for Question 25

Terms of a pension plan:

Normal retirement benefit: $1,000 per month.

Normal form of payment: Life annuity.

Data for participant Smith:

Date of birth: 1/1/34.
Date of retirement: 1/1/99.
Date of first payment: 1/1/99.

Valuation interest rate: 7% per year, compounded annually.

Selected values:

| x | $l_x$ | $D_x$ | $N_x$ | $S_x$ | $\ddot{a}_x^{(12)}$ |
|----|-----------|--------|---------|-----------|------|
| 65 | 7,673,270 | 94,414 | 868,053 | 6,655,129 | 8.74 |
| 66 | 7,500,146 | 86,246 | 773,639 | 5,787,076 | 8.51 |
| 67 | 7,313,790 | 78,601 | 687,393 | 5,013,437 | 8.29 |
| 68 | 7,114,621 | 71,459 | 608,792 | 4,326,044 | 8.06 |
| 69 | 6,903,786 | 64,805 | 537,333 | 3,717,252 | 7.83 |

Smith elects an actuarially equivalent life annuity equal to $P per month, increasing by 3% of the prior year's monthly annuity amount on 1/1 of each of the next three years, and level thereafter.

Question 25

In what range is $P?

[A]   Less than $923

[B]   $923 but less than $933

[C]   $933 but less than $943

[D]   $943 but less than $953

[E]   $953 or more

1999

# ANSWER KEY

## MAY 1999 COURSE 141 (EA-1) SEGMENT A

1. B
2. B
3. A
4. C
5. D

6. D
7. B
8. A
9. E
10. B

11. D
12. B
13. C
14. D
15. D

16. C
17. D
18. D
19. D
20. C

21. B
22. D
23. B
24. B
25. B