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Contents

Lesson 6

Put-Call Parity

Reading: *Derivatives Markets* 9.1–9.2

In this lesson and the next, rather than presenting a model for stocks or other assets so that we can price options, we discuss general properties that are true regardless of model. In this lesson, we discuss the relationship between the premium of a call and the premium of a put.

Suppose you bought a European call option and sold a European put option, both having the same underlying asset, the same strike, and the same time to expiry. *In this entire section, we will deal only with European options, not American ones,* so henceforth "European" should be understood. As above, let the value of the underlying asset be S_t at time t. You would then pay $C(K,T)-P(K,T)$ at time 0. Interestingly, an equivalent result can be achieved without using options at all! Do you see how?

The point is that at time T, one of the two options is sure to be exercised, unless the price of the asset at time T happens to exactly equal the strike price($S_T = K$), in which case both options are worthless. Whichever option is exercised, you pay K and receive the underlying asset:

- If $S_T > K$, you exercise the call option you bought. You pay K and receive the asset.
- If $K > S_T$, the counterparty exercises the put option you sold. You receive the asset and pay K.
- If $S_T = K$, it doesn't matter whether you have K or the underlying asset.

Therefore, there are two ways to receive S_T at time T:

1. Buy a call option and sell a put option at time 0, and pay K at time T.
2. Enter a forward agreement to buy S_T, and at time T pay $F_{0,T}$, the price of the forward agreement.

By the no-arbitrage principle, these two ways must cost the same. Discounting to time 0, this means

$$C(K,T) - P(K,T) + Ke^{-rT} = F_{0,T}e^{-rT}$$

or

Put-Call Parity

$$\boxed{C(K,T) - P(K,T) = e^{-rT}(F_{0,T} - K)} \tag{6.1}$$

Here's another way to derive the equation. Suppose you would like to have the maximum of S_T and K at time T. What can you buy now that will result in having this maximum? There are two choices:

- You can buy a time-T forward on the asset, and buy a put option with expiry T and strike price K. The forward price is $F_{0,T}$, but since you pay at time 0, you pay $e^{-rT}F_{0,T}$ for the forward.
- You can buy a risk-free investment maturing for K at time T, and buy a call option on the asset with expiry T and strike price K. The cost of the risk-free investment is Ke^{-rT}.

Both methods must have the same cost, so

$$e^{-rT}F_{0,T} + P(S,K,T) = Ke^{-rT} + C(S,K,T)$$

which is the same as equation (6.1).

The Put-Call Parity equation lets you price a put once you know the price of a call.

Let's go through specific examples.

6.1 Stock put-call parity

For a nondividend paying stock, the forward price is $F_{0,T} = S_0 e^{rT}$. Equation (6.1) becomes

$$C(K,T) - P(K,T) = S_0 - Ke^{-rT}$$

The right hand side is the present value of the asset minus the present value of the strike.

Example 6A A nondividend paying stock has a price of 40. A European call option allows buying the stock for 45 at the end of 9 months. The continuously compounded risk-free rate is 5%. The premium of the call option is 2.84.

Determine the premium of a European put option allowing selling the stock for 45 at the end of 9 months.

Answer: Don't forget that 5% is a continuously compounded rate.

$$\begin{aligned} C(K,T) - P(K,T) &= S_0 - Ke^{-rT} \\ 2.84 - P(K,T) &= 40 - 45e^{-(0.05)(0.75)} \\ 2.84 - P(K,T) &= 40 - 45(0.963194) = -3.34375 \\ P(K,T) &= 2.84 + 3.34375 = \boxed{\mathbf{6.18375}} \end{aligned}$$ □

A convenient way to express put-call parity is with prepaid forwards. Using prepaid forwards, the put-call parity formula becomes

$$\boxed{C(K,T) - P(K,T) = F^P_{0,T} - Ke^{-rT}} \tag{6.2}$$

Using this, let's discuss a dividend paying stock. If a stock pays discrete dividends, the formula becomes

$$C(K,T) - P(K,T) = S_0 - \mathrm{PV}_{0,T}(\mathrm{Divs}) - Ke^{-rT} \tag{6.3}$$

Example 6B A stock's price is 45. The stock will pay a dividend of 1 after 2 months. A European put option with a strike of 42 and an expiry date of 3 months has a premium of 2.71. The continuously compounded risk-free rate is 5%.

Determine the premium of a European call option on the stock with the same strike and expiry.

Answer: Using equation (6.3),

$$C(K,T) - P(K,T) = S_0 - \mathrm{PV}_{0,T}(\mathrm{Divs}) - Ke^{-rT}$$

$\mathrm{PV}_{0,T}(\mathrm{Divs})$, the present value of dividends, is the present value of the dividend of 1 discounted 2 months at 5%, or $(1)e^{-0.05/6}$.

$$\begin{aligned} C(42, 0.25) - 2.71 &= 45 - (1)e^{-0.05/6} - 42e^{-0.05(0.25)} \\ &= 45 - 0.991701 - 42(0.987578) = 2.5300 \\ C(42, 0.25) &= 2.71 + 2.5300 = \boxed{\mathbf{5.2400}} \end{aligned}$$ □

Quiz 6-1 A stock's price is 50. The stock will pay a dividend of 2 after 4 months. A European call option with a strike of 50 and an expiry date of 6 months has a premium of 1.62. The continuously compounded risk-free rate is 4%.

Determine the premium of a European put option on the stock with the same strike and expiry.

Now let's consider a stock with continuous dividends at rate δ. Using prepaid forwards, put-call parity becomes

$$C(K,T) - P(K,T) = S_0 e^{-\delta T} - Ke^{-rT} \tag{6.4}$$

Example 6C You are given:

(i) A stock's price is 40.
(ii) The continuously compounded risk-free rate is 8%.
(iii) The stock's continuous dividend rate is 2%.

A European 1-year call option with a strike of 50 costs 2.34.

Determine the premium for a European 1-year put option with a strike of 50.

Answer: Using equation (6.4),

$$\begin{aligned} C(K,T) - P(K,T) &= S_0 e^{-\delta T} - Ke^{-rT} \\ 2.34 - P(K,T) &= 40e^{-0.02} - 50e^{-0.08} \\ &= 40(0.9801987) - 50(0.9231163) = -6.94787 \\ P(K,T) &= 2.34 + 6.94787 = \boxed{9.28787} \end{aligned}$$

□

Quiz 6-2 You are given:

(i) A stock's price is 57.
(ii) The continuously compounded risk-free rate is 5%.
(iii) The stock's continuous dividend rate is 3%.

A European 3-month put option with a strike of 55 costs 4.46.

Determine the premium of a European 3-month call option with a strike of 55.

6.2 Synthetic stocks and Treasuries

Since the put-call parity equation includes terms for stock (S_0) and cash (K), we can create a synthetic stock with an appropriate combination of options and lending. With continuous dividends, the formula is

$$\begin{aligned} C(K,T) - P(K,T) &= S_0 e^{-\delta T} - Ke^{-rT} \\ S_0 &= \left(C(K,T) - P(K,T) + Ke^{-rT}\right)e^{\delta T} \end{aligned} \tag{6.5}$$

For example, suppose the risk-free rate is 5%. We want to create an investment equivalent to a stock with continuous dividend rate of 2%. We can use any strike price and any expiry; let's say 40 and 1 year. We have

$$S_0 = \left(C(40,1) - P(40,1) + 40e^{-0.05}\right)e^{0.02}$$

So we buy $e^{0.02} = 1.02020$ call options and sell 1.02020 put options, and buy a Treasury for 38.8178. At the end of a year, the Treasury will be worth $38.8178e^{0.05} = 40.808$. An option will be exercised, so we will pay $40(1.02020) = 40.808$ and get 1.02020 shares of the stock, which is equivalent to buying 1 share of the stock originally and reinvesting the dividends.

If dividends are discrete, then they are assumed to be fixed in advance, and the formula becomes

$$\begin{aligned} C(K,T) - P(K,T) &= S_0 - \text{PV(dividends)} - Ke^{-rT} \\ S_0 &= C(K,T) - P(K,T) + \underbrace{\text{PV(dividends)} + Ke^{-rT}}_{\text{amount to lend}} \end{aligned} \tag{6.6}$$

For example, suppose the risk-free rate is 5%, the stock is 40, and the period is 1 year. The dividends are 0.5 apiece at the end of 3 months and at the end of 9 months. Then their present value is

$$0.5e^{-0.05(0.25)} + 0.5e^{-0.05(0.75)} = 0.97539$$

To create a synthetic stock, we buy a call, sell a put, and lend $0.97539 + 40e^{-0.05} = 39.0246$. At the end of the year, we'll have 40 plus the accumulated value of the dividends. One of the options will be exercised, so the 40 will be exchanged for one share of the stock.

To create a synthetic Treasury[1], we rearrange the equation as follows:

$$\begin{aligned} C(K,T) - P(K,T) &= S_0e^{-\delta T} - Ke^{-rT} \\ Ke^{-rT} &= S_0e^{-\delta T} - C(K,T) + P(K,T) \end{aligned} \tag{6.7}$$

We buy $e^{-\delta T}$ shares of the stock and a put option and sell a call option. Using $K = 40$, $r = 0.05$, $\delta = 0.02$, and 1 year to maturity again, the total cost of this is $Ke^{-rT} = 40e^{-0.05} = 38.04918$. At the end of the year, we sell the stock for 40 (since one option will be exercised). This is equivalent to investing in a one-year Treasury bill with maturity value 40.

If dividends are discrete, then they are assumed to be fixed in advance and can be combined with the strike price as follows:

$$Ke^{-rT} + \text{PV(dividends)} = S_0 - C(K,T) + P(K,T) \tag{6.8}$$

The maturity value of this Treasury is $K + \text{CumValue(dividends)}$. For example, suppose the risk-free rate is 5%, the stock is 40, and the period is 1 year. The dividends are 0.5 apiece at the end of 3 months and at the end of 9 months. Then their present value, as calculated above, is

$$0.5e^{-0.05(0.25)} + 0.5e^{-0.05(0.75)} = 0.97539$$

and their accumulated value at the end of the year is $0.97539e^{0.05} = 1.02540$. Thus if you buy a stock and a put and sell a call, both options with strike prices 40, the investment will be $Ke^{-0.05} + 0.97539 = 39.0246$ and the maturity value will be $40 + 1.02540 = 41.0254$.

Quiz 6-3 You wish to create a synthetic investment using options on a stock. The continuously compounded risk-free interest rate is 4%. The stock price is 43. You will use 6-month options with a strike of 45. The stock pays continuous dividends at a rate of 1%. The synthetic investment should duplicate 100 shares of the stock.

Determine the amount you should invest in Treasuries.

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[1]Creating a synthetic Treasury is called a *conversion*. Selling a synthetic Treasury by shorting the stock, buying a call, and selling a put, is called a *reverse conversion*.

6.3 Synthetic options

If an option is mispriced based on put-call parity, you may want to create an arbitrage.

Suppose the price of a European call based on put-call parity is C, but the price it is actually selling at is $C' < C$. (From a different perspective, this may indicate the put is mispriced, but let's assume the price of the put is correct based on some model.) You would then buy the underpriced call option and sell a synthesized call option. Since

$$C(S,K,t) = Se^{-\delta t} - Ke^{-rt} + P(S,K,t)$$

you would sell the right hand side of this equation. You'd sell $e^{-\delta t}$ shares of the underlying stock, sell a European put option with strike price K and expiry t, and buy a risk-free zero-coupon bond with a price of Ke^{-rt}, or in other words lend Ke^{-rt} at the risk-free rate. These transactions would give you $C(S,K,t)$. You'd pay C' for the option you bought, and keep the difference.

6.4 Exchange options

The options we've discussed so far involve receiving/giving a stock in return for cash. We can generalize to an option to receive a stock in return for a different stock. Let S_t be the value of the underlying asset, the one for which the option is written, and Q_t be the price of the strike asset, the one which is paid. Forwards will now have a parameter for the asset; $F_{t,T}(Q)$ will mean a forward agreement to purchase asset Q (actually, the asset with price Q_t) at time T. A superscript P will indicate a prepaid forward, as before. Calls and puts will have an extra parameter too:

- $C(S_t, Q_t, T-t)$ means a call option written at time t which lets the purchaser elect to receive S_T in return for Q_T at time T; in other words, to receive $\max(0, S_T - Q_T)$.
- $P(S_t, Q_t, T-t)$ means a put option written at time t which lets the purchaser elect to give S_T in return for Q_T; in other words, to receive $\max(0, Q_T - S_T)$.

The put-call parity equation is then

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q) \tag{6.9}$$

Exchange options are sometimes given to corporate executives. They are given a call option on the company's stock against an index. If the company's stock performs better than the index, they get compensated.

Notice how the definitions of calls and puts are mirror images. A call on one share of Ford with one share of General Motors as the strike asset is the same as a put on one share of General Motors with one share of Ford as the strike asset. In other words

$$P(S_t, Q_t, T-t) = C(Q_t, S_t, T-t)$$

and we could've written the above equation with just calls:

$$C(S_t, Q_t, T-t) - C(Q_t, S_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q)$$

Example 6D A European call option allows one to purchase 2 shares of Stock B with 1 share of Stock A at the end of a year. You are given:

(i) The continuously compounded risk-free rate is 5%.
(ii) Stock A pays dividends at a continuous rate of 2%.
(iii) Stock B pays dividends at a continuous rate of 4%.
(iv) The current price for Stock A is 70.
(v) The current price for Stock B is 30.

A European put option which allows one to sell 2 shares of Stock B for 1 share of Stock A costs 11.50.

Determine the premium of the European call option mentioned above, which allows one to purchase 2 shares of Stock B for 1 share of Stock A.

Answer: The risk-free rate is irrelevant. Stock B is the underlying asset (price S in the above notation) and Stock A is the strike asset (price Q in the above notation). By equation (6.9),

$$\begin{aligned}
C(S,Q,1) &= 11.50 + F^P_{0,T}(S) - F^P_{0,T}(Q)\\
F^P_{0,T}(S) &= S_0 e^{-\delta_S T} = (2)(30)e^{-0.04} = 57.64737\\
F^P_{0,T}(Q) &= Q_0 e^{-\delta_Q T} = (70)e^{-0.02} = 68.61391\\
C(S,Q,1) &= 11.50 + 57.64737 - 68.61391 = \boxed{\mathbf{0.5335}}
\end{aligned}$$

□

Quiz 6-4 In the situation of Example 6D, determine the premium of a European call option which allows one to buy 1 share of Stock A for 2 shares of Stock B at the end of a year.

6.5 Currency options

Let $C(x_0, K, T)$ be a call option on currency with spot exchange rate[2] x_0 to purchase it at exchange rate K at time T, and $P(x_0, K, T)$ the corresponding put option. Putting equation 6.2 and the last formula in Table 3.1 together, we have the following formula:

$$C(x_0, K, T) - P(x_0, K, T) = x_0 e^{-r_f T} - K e^{-r_d T} \tag{6.10}$$

where r_f is the "foreign" risk-free rate for the currency which is playing the role of a stock (the one which can be purchased for a call option or the one that can be sold for a put option) and r_d is the "domestic" risk-free rate which is playing the role of cash in a stock option (the one which the option owner pays in a call option and the one which the option owner receives in a put option).

Example 6E You are given:

(i) The spot exchange rate for dollars to pounds is 1.4\$/£.
(ii) The continuously compounded risk-free rate for dollars is 5%.
(iii) The continuously compounded risk-free rate for pounds is 8%.

A 9-month European put option allows selling £1 at the rate of \$1.50/£. A 9-month dollar denominated call option with the same strike costs \$0.0223.

Determine the premium of the 9-month dollar denominated put option.

Answer: The prepaid forward price for pounds is

$$x_0 e^{-r_f T} = 1.4e^{-0.08(0.75)} = 1.31847$$

The prepaid forward for the strike asset (dollars) is

$$K e^{-r_d T} = 1.5e^{-0.05(0.75)} = 1.44479$$

Thus

$$\begin{aligned}
C(x_0, K, T) - P(x_0, K, T) &= 1.31847 - 1.44479 = -0.12632\\
0.0223 - P(1.4, 1.5, 0.75) &= -0.12632\\
P(1.4, 1.5, 0.75) &= 0.0223 + 0.12632 = \boxed{\mathbf{0.14862}}
\end{aligned}$$

□

[2]The word "spot", as in spot exchange rate or spot price or spot interest rate, refers to the current rates, in contrast to *forward* rates.

Quiz 6-5 You are given:

(i) The spot exchange rate for yen to dollars is 90¥/$.

(ii) The continuously compounded risk-free rate for dollars is 5%.

(iii) The continuously compounded risk-free rate for yen is 1%.

A 6-month yen-denominated European call option on dollars has a strike price of 92¥/$ and costs ¥0.75.

Calculate the premium of a 6-month yen-denominated European put option on dollars having a strike price of 92¥/$.

A call to purchase pounds with dollars is equivalent to a put to sell dollars for pounds. However, the units are different. Let's see how to translate units.

EXAMPLE 6F The spot exchange rate for dollars into euros is $1.05/€. A 6-month dollar denominated call option to buy one euro at strike price $1.1/€1 costs $0.04.

Determine the premium of the corresponding euro-denominated put option to sell one dollar for euros at the corresponding strike price.

ANSWER: To sell 1 dollar, the corresponding exchange rate would be $1/€$\frac{1}{1.1}$, so the euro-denominated strike price is $\frac{1}{1.1} = 0.9091$€/$. Since we're in effect buying 0.9091 of the dollar-denominated call option, the premium in dollars is ($0.04)(0.9091) = $0.03636. Dividing by the spot rate, the premium in euros is $\frac{0.03636}{1.05} = \boxed{\textbf{€0.03463}}$ □

Let's generalize the example. Let the domestic currency be the one the option is denominated in, the one in which the price is expressed. Let the foreign currency be the underlying asset of the option. Consider a call option, with the following parameters:

1. The spot rate is x_0 units of domestic currency.
2. The strike price is K units of domestic currency.

Then the call premium is $C(x_0, K, T)$ units of domestic currency. The call option allows one to buy 1 unit of foreign currency for K units of domestic currency. To identify the currency used to price an option, we'll use d for domestic and f for foreign. Our call premium is $C_d(x_0, K, T)$.

Now let's create an equivalent put option. This will allow one to sell K units of domestic currency for 1 unit of foreign currency. But a single unit of a put option allows selling 1 unit, so one unit of the equivalent put option must allow selling 1 unit of domestic currency for $1/K$ units of foreign currency. Moreover, the spot price of domestic currency is $1/x_0$ in foreign currency. So we need $KP_d(1/x_0, 1/K, t)$ in domestic currency to equate to $C_d(x_0, K, T)$ in domestic currency.

$$KP_d\left(\frac{1}{x_0}, \frac{1}{K}, T\right) = C_d(x_0, K, T)$$

Since the put option's price should be expressed in the foreign currency, the left side must be multiplied by x_0, resulting in

$$Kx_0P_f\left(\frac{1}{x_0}, \frac{1}{K}, T\right) = C_d(x_0, K, T)$$

where P_f is the price of the put option in the foreign currency.

Note that if settlement is through cash rather than through actual exchange of currencies, then the put options $Kx_0P_f(1/x_0, 1/K, T)$ may not have the same payoff as the call option $C_d(x_0, K, T)$. The put options $Kx_0P_f(1/x_0, 1/K, T)$ pay off in the foreign currency while the call option $C_d(x_0, K, T)$ pays off in the domestic currency. The payoffs are equal based on exchange rate x_0, but the exchange rate x_T at time T may be different from x_0.

Quiz 6-6 The spot rate for yen denominated in pounds sterling is 0.005£/¥. A 3-month pound-denominated put option has strike 0.0048£/¥ and costs £0.0002.

Determine the premium in yen for an equivalent 3-month yen-denominated call option with a strike of ¥$208\frac{1}{3}$.

Exercises

Put-call parity for stock options

6.1. [**CAS8-S03:18a**] A four-month European call option with a strike price of 60 is selling for 5. The price of the underlying stock is 61, and the annual continuously compounded risk-free rate is 12%. The stock pays no dividends.

Calculate the value of a four-month European put option with a strike price of 60.

6.2. For a nondividend paying stock, you are given:

(i) Its current price is 30.
(ii) A European call option on the stock with one year to expiration and strike price 25 costs 8.05.
(iii) The continuously compounded risk-free interest rate is 0.05.

Determine the premium of a 1-year European put option on the stock with strike 25.

6.3. A nondividend paying stock has price 30. You are given:

(i) The continuously compounded risk-free interest rate is 5%.
(ii) A 6-month European call option on the stock costs 3.10.
(iii) A 6-month European put option on the stock with the same strike price as the call option costs 5.00.

Determine the strike price.

6.4. A stock pays continuous dividends proportional to its price at rate δ. You are given:

(i) The stock price is 40.
(ii) The continuously compounded risk-free interest rate is 4%.
(iii) A 3-month European call option on the stock with strike 40 costs 4.10.
(iv) A 3-month European put option on the stock with strike 40 costs 3.91.

Determine δ.

6.5. For a stock paying continuous dividends proportional to its price at rate $\delta = 0.02$, you are given:

(i) The continuously compounded risk-free interest rate is 3%.
(ii) A 6-month European call option with strike 40 costs 4.10.
(iii) A 6-month European put option with strike 40 costs 3.20.

Determine the current price of the stock.

Exercises continue on the next page . . .

6.6. A stock's price is 45. Dividends of 2 are payable quarterly, with the next dividend payable at the end of one month. You are given:

(i) The continuously compounded risk-free interest rate is 6%.
(ii) A 3-month European put option with strike 50 costs 7.32.

Determine the premium of a 3-month European call option on the stock with strike 50.

6.7. A dividend paying stock has price 50. You are given:

(i) The continuously compounded risk-free interest rate is 6%.
(ii) A 6-month European call option on the stock with strike 50 costs 2.30.
(iii) A 6-month European put option on the stock with strike 50 costs 1.30.

Determine the present value of dividends paid over the next 6 months on the stock.

6.8. Consider European options on a stock expiring at time t. Let $P(K)$ be a put option with strike price K, and $C(K)$ be a call option with strike price K. You are given

(i) $P(50) - C(55) = -2$
(ii) $P(55) - C(60) = 3$
(iii) $P(60) - C(50) = 14$

Determine $C(60) - P(50)$.

6.9. [**CAS8-S00:26**] You are given the following:

(i) Stock price = \$50
(ii) The risk-free interest rate is a constant annual 8%, compounded continuously
(iii) The price of a 6-month European call option with an exercise price of \$48 is \$5.
(iv) The price of a 6-month European put option with an exercise price of \$48 is \$3.
(v) The stock pays no dividends

There is an arbitrage opportunity involving buying or selling one share of stock and buying or selling puts and calls.

Calculate the profit after 6 months from this strategy.

6.10. [**Introductory Derivatives Sample Question 2**] You are given the following:

(i) The current price to buy one share of XYZ stock is 500.
(ii) The stock does not pay dividends.
(iii) The annual risk-free interest rate, compounded continuously, is 6%.
(iv) A European call option on one share of XYZ stock with a strike price of K that expires in one year costs 66.59.
(v) A European put option on one share of XYZ stock with a strike price of K that expires in one year costs 18.64.

Using put-call parity, calculate the strike price, K.

(A) 449 (B) 452 (C) 480 (D) 559 (E) 582

Exercises continue on the next page . . .

6.11. [**Introductory Derivatives Sample Question 5**] The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000.
- The PS index does not pay dividends.

Sam wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of 1,025.

The annual effective risk-free interest rate is 5%.

Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.

(A) Buy the put and sell the call, receive 23.81
(B) Buy the put and sell the call, spend 23.81
(C) Buy the put and sell the call, no cost
(D) Buy the call and sell the put, receive 23.81
(E) Buy the call and sell the put, spend 23.81

6.12. [**Introductory Derivatives Sample Question 14**] The current price of a non-dividend paying stock is 40 and the continuously compounded annual risk-free rate of return is 8%. You are given that the price of a 35-strike call option is 3.35 higher than the price of a 40-strike call option, where both options expire in 3 months.

Calculate the amount by which the price of an otherwise equivalent 40-strike put option exceeds the price of an otherwise equivalent 35-strike put option.

(A) 1.55 (B) 1.65 (C) 1.75 (D) 3.25 (E) 3.35

6.13. [**Introductory Derivatives Sample Question 41**] XYZ stock pays no dividends and its current price is 100.

Assume the put, the call and the forward on XYZ stock are available and are priced so there are no arbitrage opportunities. Also, assume there are no transaction costs.

The current risk-free annual effective interest rate is 1%.

Determine which of the following strategies currently has the highest net premium.

(A) Long a six-month 100-strike put and short a six-month 100-strike call
(B) Long a six-month forward on the stock
(C) Long a six-month 101-strike put and short a six-month 101-strike call
(D) Short a six-month forward on the stock
(E) Long a six-month 105-strike put and short a six-month 105-strike call

6.14. [**Introductory Derivatives Sample Question 40**] An investor is analyzing the costs of two-year, European options for aluminum and zinc at a particular strike price.

For each ton of aluminum, the two-year forward price is 1400, a call option costs 700, and a put option costs 550.

For each ton of zinc, the two-year forward price is 1600 and a put option costs 550.

The risk-free annual effective interest rate is a constant 6%.

Calculate the cost of a call option per ton of zinc.

(A) 522 (B) 800 (C) 878 (D) 900 (E) 1231

Exercises continue on the next page . . .

6.15. [**Introductory Derivatives Sample Question 53**] For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year 400-strike European call option costs 110.

The annual risk-free force of interest is a constant 6.5%.

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

(A) 10.00 (B) 32.89 (C) 118.42 (D) 187.11 (E) 210.00

6.16. [**Introductory Derivatives Sample Question 65**] Assume that a single stock is the underlying asset for a forward contract, a K-strike call option, and a K-strike put option.

Assume also that all three derivatives are evaluated at the same point in time.

Which of the following formulas represents put-call parity?

(A) Call Premium − Put Premium = Present Value(Forward Price − K)
(B) Call Premium − Put Premium = Present Value(Forward Price)
(C) Put Premium − Call Premium = 0
(D) Put Premium − Call Premium = Present Value(Forward Price − K)
(E) Put Premium − Call Premium = Present Value(Forward Price)

6.17. [**Introductory Derivatives Sample Question 72**] CornGrower is going to sell corn in one year. In order to lock in a fixed selling price, CornGrower buys a put option and sells a call option on each bushel, each with the same strike price and the same one-year expiration date. The current price of corn is 3.59 per bushel, and the net premium that CornGrower pays now to lock in the future price is 0.10 per bushel.

The continuously compounded risk-free interest rate is 4%.

Calculate the fixed selling price per bushel one year from now.

(A) 3.49 (B) 3.63 (C) 3.69 (D) 3.74 (E) 3.84

Synthetic assets

6.18. You are given:

(i) The price of a stock is 43.00.
(ii) The continuously compounded risk-free interest rate is 5%.
(iii) The stock pays a dividend of 1.00 three months from now.
(iv) A 3-month European call option on the stock with strike 44.00 costs 1.90.

You wish to create this stock synthetically, using a combination of 44-strike options expiring in 3 months and lending.

Determine the amount of money you should lend.

6.19. You are given:

(i) The price of a stock is 95.00.
(ii) The continuously compounded risk-free rate is 6%.
(iii) The stock pays quarterly dividends of 0.80, with the next dividend payable in 1 month.

You wish to create this stock synthetically, using 1-year European call and put options with strike price K, and lending 96.35.

Determine the strike price of the options.

Exercises continue on the next page . . .

6.20. You are given:

(i) A stock index is 22.00.
(ii) The continuous dividend rate of the index is 2%.
(iii) The continuously compounded risk-free interest rate is 5%.
(iv) A 3-month European call option on the index with strike 21.00 costs 1.90.
(v) A 3-month European put option on the index with strike 21.00 costs 0.75.

You wish to create an equivalent synthetic stock index using a combination of options and lending.

Determine the amount of money you should lend.

6.21. You are given:

(i) The stock price is 40.
(ii) The stock pays continuous dividends proportional to its price at a rate of 1%.
(iii) The continuously compounded risk-free interest rate is 4%.
(iv) A 182-day European put option on the stock with strike 50 costs 11.00.

You wish to create a synthetic 182-day Treasury bill with maturity value 10,000.

Determine the number of shares of the stock you should purchase.

6.22. You wish to create a synthetic 182-day Treasury bill with maturity value 10,000. You are given:

(i) The stock price is 40.
(ii) The stock pays continuous dividends proportional to its price at a rate of 2%.
(iii) A 182-day European put option on the stock with strike price K costs 0.80.
(iv) A 182-day European call option on the stock with strike price K costs 5.20.
(v) The continuously compounded risk-free interest rate is 5%.

Determine the number of shares of the stock you should purchase.

6.23. You are given:

(i) The price of a stock is 100.
(ii) The stock pays discrete dividends of 2 per quarter, with the first dividend 3 months from now.
(iii) The continuously compounded risk-free interest rate is 4%.

You wish to create a synthetic 182-day Treasury bill with maturity value 10,000, using a combination of the stock and 6-month European put and call options on the stock with strike price 95.

Determine the number of shares of the stock you should purchase.

Put-call parity for exchange options

6.24. For two stocks, S_1 and S_2:

(i) The price of S_1 is 30.
(ii) S_1 pays continuous dividends proportional to its price. The dividend yield is 2%.
(iii) The price of S_2 is 75.
(iv) S_2 pays continuous dividends proportional to its price. The dividend yield is 5%.
(v) A 1-year call option to receive a share of S_2 in exchange for 2.5 shares of S_1 costs 2.50.

Determine the premium of a 1-year call option to receive 1 share of S_1 in exchange for 0.4 shares of S_2.

Exercises continue on the next page . . .

6.25. S_1 is a stock with price 30 and quarterly dividends of 0.25. The next dividend is payable in 3 months.

S_2 is a nondividend paying stock with price 40.

The continuously compounded risk-free interest rate is 5%.

Let x be the premium of an option to give S_1 in exchange for receiving S_2 at the end of 6 months, and let y be the premium of an option to give S_2 in exchange for receiving S_1 at the end of 6 months.

Determine $x - y$.

6.26. For the stocks of Sohitu Autos and Flashy Autos, you are given:

(i) The price of one share of Sohitu is 180.
(ii) The price of one share of Flashy is 90.
(iii) Sohitu pays quarterly dividends of 3 on Feb. 15, May 15, Aug. 15, and Nov. 15 of each year.
(iv) Flashy pays quarterly dividends of 1 on Jan. 31, Apr. 30, July 31, and Oct. 31 of each year.
(v) The continuously compounded risk-free interest rate is 0.06.
(vi) On Dec. 31, an option expiring in 6 months to get x shares of Flashy for 1 share of Sohitu costs 4.60.
(vii) On Dec. 31, an option expiring in 6 months to get 1 share of Sohitu for x shares of Flashy costs 7.04.

Determine x.

6.27. Stock A is a nondividend paying stock. Its price is 100.

Stock B pays continuous dividends proportional to its price. The dividend yield is 0.03. Its price is 60.

An option expiring in one year to buy x shares of A for 1 share of B costs 2.39.

An option expiring in one year to buy $1/x$ shares of B for 1 share of A costs 2.74.

Determine x.

Put-call parity for currency options

6.28. You are given

(i) The spot exchange rate is 95¥/\$1.
(ii) The continuously compounded risk-free rate in yen is 1%.
(iii) The continuously compounded risk-free rate in dollars is 5%.
(iv) A 1-year dollar denominated European call option on yen with strike \$0.01 costs \$0.0011.

Determine the premium of a 1-year dollar denominated European put option on yen with strike \$0.01.

Exercises continue on the next page . . .

Use the following information for questions 6.29 and 6.30:

You are given:

(i) The spot exchange rate is 1.5\$/£.
(ii) The continuously compounded risk-free rate in dollars is 6%.
(iii) The continuously compounded risk-free rate in pounds sterling is 3%.
(iv) A 6-month dollar-denominated European put option on pounds with a strike of 1.5\$/£ costs \$0.03.

6.29. Determine the premium in pounds of a 6-month pound-denominated European call option on dollars with a strike of (1/1.5)£/\$.

6.30. Determine the premium in pounds of a 6-month pound-denominated European put option on dollars with a strike of (1/1.5)£/\$.

6.31. The spot exchange rate of dollars for euros is 1.2\$/€. A dollar-denominated put option on euros has strike price \$1.3.

Determine the strike price of the corresponding euro-denominated call option to pay a certain number of euros for one dollar.

6.32. You are given:

(i) The spot exchange rate of dollars for euros is 1.2\$/€.
(ii) A one-year dollar-denominated European call option on euros with strike price \$1.3 costs 0.05.
(iii) The continuously compounded risk-free interest rate for dollars is 5%.
(iv) A one-year dollar-denominated European put option on euros with strike price \$1.3 costs 0.20.

Determine the continuously compounded risk-free interest rate for euros.

6.33. You are given:

(i) The spot exchange rate of yen for euros is 110¥/€.
(ii) The continuously compounded risk-free rate for yen is 2%.
(iii) The continuously compounded risk-free rate for euros is 4%.
(iv) A one year yen-denominated call on euros costs ¥3.
(v) A one year yen-denominated put on euros with the same strike price as the call costs ¥2.

Determine the strike price in yen.

6.34. You are given:

(i) The continuously compounded risk-free interest rate for dollars is 4%.
(ii) The continuously compounded risk-free interest rate for pounds is 6%.
(iii) A 1-year dollar-denominated European call option on pounds with strike price 1.6 costs \$0.05.
(iv) A 1-year dollar-denominated European put option on pounds with strike price 1.6 costs \$0.10.

Determine the spot exchange rate of dollars per pound.

Exercises continue on the next page . . .

6.35. You are given:

(i) The continuously compounded risk-free interest rate for dollars is 4%.
(ii) The continuously compounded risk-free interest rate for pounds is 6%.
(iii) A 6-month dollar-denominated European call option on pounds with strike price 1.45 costs \$0.05.
(iv) A 6-month dollar-denominated European put option on pounds with strike price 1.45 costs \$0.02.

Determine the 6-month forward exchange rate of dollars per pound.

Additional released exam questions: Sample:1, CAS3-S07:3,4,13, SOA MFE-S07:1, CAS3-F07:14,15,16,25, MFE/3F-S09:9

Solutions

6.1. By put-call parity,

$$\begin{aligned} P(61,60,1/3) &= C(61,60,1/3) + 60e^{-r(1/3)} - 61e^{-\delta(1/3)} \\ &= 5 + 60e^{-0.04} - 61 = \boxed{\mathbf{1.647}} \end{aligned}$$

6.2. We want $P(25,1)$. By put-call parity:

$$\begin{aligned} C(25,1) - P(25,1) &= S - Ke^{-r} \\ 8.05 - P(25,1) &= 30 - 25e^{-0.05} = 6.2193 \\ P(25,1) &= 8.05 - 6.2193 = \boxed{\mathbf{1.8307}} \end{aligned}$$

6.3. We need K, the strike price. By put-call parity:

$$\begin{aligned} C(K,0.5) - P(K,0.5) &= S - Ke^{-0.5r} \\ 3.10 - 5.00 &= 30 - Ke^{-0.5(0.05)} \\ -31.90 &= -Ke^{-0.025} \\ K &= 31.90e^{0.025} = \boxed{\mathbf{32.7076}} \end{aligned}$$

6.4. By put-call parity,

$$\begin{aligned} C(40,0.25) - P(40,0.25) &= Se^{-0.25\delta} - Ke^{-0.25r} \\ 4.10 - 3.91 &= 40e^{-0.25\delta} - 40e^{-0.01} \\ &= 40e^{-0.25\delta} - 39.6020 \\ 0.19 + 39.6020 &= 40e^{-0.25\delta} \\ 39.7920 &= 40e^{-0.25\delta} \\ e^{-0.25\delta} &= \frac{39.7920}{40} = 0.9948 \\ 0.25\delta &= -\ln 0.9948 = 0.005214 \\ \delta &= \frac{0.005214}{0.25} = \boxed{\mathbf{0.02086}} \end{aligned}$$

6.5. By put-call parity,

$$\begin{aligned}C(40,0.5)-P(40,0.5)&=Se^{-0.5\delta}-Ke^{-0.5r}\\4.10-3.20&=Se^{-0.01}-40e^{-0.015}\\Se^{-0.01}&=0.9+39.40448=40.30448\\S&=40.30448e^{0.01}=\boxed{\mathbf{40.7095}}\end{aligned}$$

6.6. The value of the prepaid forward on the stock is

$$F^P_{0.25}=S-\text{PV(Div)}=45-2e^{-0.06(1/12)}=45-1.9900=43.0100$$

By put-call parity,

$$\begin{aligned}C(50,0.25)-P(50,0.25)&=43.0100-Ke^{-0.25(0.06)}\\C(50,0.25)-7.32&=43.0100-49.2556=-6.2456\\C(50,0.25)&=7.32-6.2456=\boxed{\mathbf{1.0744}}\end{aligned}$$

6.7. By put-call parity,

$$\begin{aligned}C(50,0.5)-P(50,0.5)&=S-\text{PV(Divs)}-Ke^{-r(0.5)}\\2.30-1.30&=50-\text{PV(Divs)}-50e^{-0.03}\\\text{PV(Divs)}&=50-2.30+1.30-50(0.970446)=\boxed{\mathbf{0.4777}}\end{aligned}$$

6.8. By put-call parity, adding up the three given statements

$$P(50)-C(50)+P(55)-C(55)+P(60)-C(60)=(50+55+60)e^{-rt}-3Se^{-\delta t}=-2+3+14=15$$

Then $P(60)-C(60)+P(50)-C(50)=110e^{-rt}-2Se^{-\delta t}=(2/3)(15)=10$. Since $P(60)-C(50)=14$, it follows that $P(50)-C(60)=10-14=-4$, and $C(60)-P(50)=\boxed{\mathbf{4}}$.

6.9. First use put-call parity to determine whether the put is underpriced or overpriced relative to the call.

$$\begin{aligned}P(50,48,0.5)&=C(50,48,0.5)+48e^{-rt}-50\\&=5+48e^{-0.04}-50=1.1179\end{aligned}$$

Since the put has price 3, it is overpriced relative to the call. This means you buy a call and sell a put. In 6 months, you must buy a share of stock, so sell one short right now. The cash flow of this strategy is

Short one share of stock	50
Buy a call	−5
Sell a put	3
	48

After 6 months, 48 will grow to $48e^{0.04}=49.96$ and you will pay 48 for the stock, for a net gain of $49.959-48=\boxed{\mathbf{1.959}}$.

6.10.

$$\begin{aligned}P-C&=Ke^{-rt}-Se^{-\delta t}\\18.64-66.59&=Ke^{-0.06}-500\\K&=(500+18.64-66.59)e^{0.06}=\boxed{\mathbf{480.00}}\end{aligned}$$

6.11. By buying a 1,025-call and selling a 1,025-put, there will be a definite purchase of the index for 1,025 in one year. By put-call parity, the cost $C-P$ is $S-Ke^{-rt}=1000-1025/1.05=\boxed{\mathbf{23.81}}$. **(E)**

6.12. By put-call parity, the excess of $P(40)-C(40)$ over $P(35)-C(35)$ is $(40-35)e^{-rt}=5e^{-0.02}$. Therefore

$$P(40)-P(35)=5e^{-0.02}+C(40)-C(35)=4.90-3.35=\boxed{\mathbf{1.55}}\qquad\textbf{(A)}$$

6.13. By put-call parity, (A),(C), and (E) are equivalent to $K/(1+i)^t-S$ with different Ks. The highest premium is from the highest K. For $K=105$, we get $105/1.01^{0.5}-100>0$. Forwards have no premium. **(E)**

6.14. The prepaid forward price is e^{-rt}, here $1/1.06^2$, times the forward price. By put-call parity for aluminum,

$$\begin{aligned}700-550&=\frac{1400}{1.06^2}-Kv^2\\ Kv^2&=1096.00\end{aligned}$$

Then by put-call parity for zinc,

$$\begin{aligned}C-550&=\frac{1600}{1.06^2}-1096.00\\ C&=\boxed{\mathbf{878.00}}\,\textbf{(C)}\end{aligned}$$

6.15. The four-year prepaid forward price is $300e^{-0.065(4)}=231.32$. By put-call parity,

$$\begin{aligned}P-C&=Ke^{-rt}-F^P(S)\\ P-110&=400e^{-0.26}-231.32\\ P&=110+400e^{-0.26}-231.32=\boxed{\mathbf{187.11}}\qquad\textbf{(D)}\end{aligned}$$

6.16. **(A)** is the correct formula, since a call minus a put is equivalent to a forward on the underlying asset, and if one will pay K at maturity, one should pay the present value of the excess of the forward price over K right now.

6.17. CornGrower invested 3.69 (current price of corn plus price of options) and thus should receive $3.69e^{0.04}=\boxed{\mathbf{3.8406}}$ at the end of the year if there is no arbitrage. **(E)**

6.18. By put-call parity,

$$\begin{aligned}C(S,44,0.25)-P(S,44,0.25)&=S-\mathrm{PV(Divs)}-Ke^{-0.05(0.25)}\\ S&=C(S,44,0.25)-P(S,44,0.25)+Ke^{-0.0125}+\mathrm{PV(Divs)}\\ &=C(S,44,0.25)-P(S,44,0.25)+44e^{-0.0125}+e^{-0.0125}\end{aligned}$$

So the amount to lend is $45e^{-0.0125}=\boxed{\mathbf{44.4410}}$.

6.19. The present value of the dividends is $0.8(e^{-0.005}+e^{-0.02}+e^{-0.035}+e^{-0.05})=3.1136$. By formula (6.6), the amount to lend is $Ke^{-rT}+PV(\text{dividends})$, or

$$\begin{aligned}Ke^{-0.06}+3.1136&=96.35\\ K=(96.35-3.1136)e^{0.06}&=\boxed{\mathbf{99.00}}\end{aligned}$$

We did not need the stock price for this exercise.

6.20. Did you verify that these option prices satisfy put-call parity? Not that you have to for this exercise. By put-call parity,

$$\begin{aligned}
C(S,21,0.25)-P(S,21,0.25) &= Se^{-(0.02)(0.25)}-Ke^{-(0.05)(0.25)}\\
S &= \frac{C(S,21,0.25)-P(S,21,0.25)+21e^{-0.0125}}{e^{-0.005}}\\
&= \big(C(S,21,0.25)-P(S,21,0.25)\big)e^{0.005}+21e^{-0.0075}
\end{aligned}$$

Thus we need to lend $21e^{-0.0075}=\boxed{\mathbf{20.84}}$. It can then be verified that buying $e^{0.005}$ calls and selling $e^{0.005}$ puts costs 1.16 for a total investment of 22, the price of the index.

6.21. By put-call parity,

$$\begin{aligned}
P(S,50,0.5)-C(S,50,0.5) &= Ke^{-0.04(0.5)}-Se^{-0.01(0.5)}\\
Ke^{-0.04(0.5)} &= P(S,50,0.5)-C(S,50,0.5)+Se^{-0.005}
\end{aligned}$$

Since we want a maturity value of 10,000, we need the left hand side to be $10,000e^{-0.02}$, so we multiply the equation by $\frac{10{,}000}{K}=200$. Thus we need to buy $200e^{-0.005}=\boxed{\mathbf{199.0025}}$ shares of stock.

6.22. We will back out the strike price K using put-call parity.

$$\begin{aligned}
P(S,K,0.5)-C(S,K,0.5) &= Ke^{-0.05(0.5)}-Se^{-0.02(0.5)}\\
0.80-5.20 &= Ke^{-0.025}-40e^{-0.01}\\
-4.40 &= Ke^{-0.025}-40e^{-0.01}\\
K &= \left(40e^{-0.01}-4.40\right)e^{0.025}\\
&= 40e^{0.015}-4.40e^{0.025}\\
&= 40.60452-4.51139=36.0931
\end{aligned}$$

By put-call parity (using the 3rd equation above)

$$Ke^{-0.025}=Se^{-0.01}+P(S,K,0.5)-C(S,K,0.5)$$

We multiply by $\frac{10{,}000}{K}$ to get a maturity value of 10,000. The number of shares of stock needed is

$$\left(\frac{10{,}000}{K}\right)e^{-0.01}=\frac{10{,}000}{36.0931}(0.99005)=\boxed{\mathbf{274.30}}$$

6.23. Using equation (6.8), the maturity value of the Treasury for every share purchased is

$$K+\text{CumValue(dividends)}=95+2e^{0.04(0.25)}+2=99.0201$$

Therefore, the number of shares of stock is $10{,}000/99.0201=\boxed{\mathbf{100.99}}$.

6.24. By put-call parity,

$$C(S_1,0.4S_2,1)-C(0.4S_2,S_1,1)=F_{0,1}^P(S_1)-F_{0,1}^P(0.4S_2)$$

A call for 0.4 shares of S_2 in return for a share of S_1 is 0.4 of the call in (v), so it is worth $(0.4)(2.50)=1.00$. Then

$$\begin{aligned}
C(S_1,0.4S_2,1)-1.00 &= 30e^{-0.02}-(0.4)(75)e^{-0.05}\\
&= 29.40596-28.53688=0.86908\\
C(S_1,0.4S_2,1) &= 0.86908+1.00=\boxed{\mathbf{1.86908}}
\end{aligned}$$

6.25. Let Divs be the dividends on S_1. By put-call parity, $x-y$ is the difference in prepaid forward prices for the two stocks. x is a call on S_2 and y is a put on S_2, so

$$x - y = S_2 - \big(S_1 - \text{PV(Divs)}\big)$$

Let's compute the present value of dividends.

$$\text{PV(Divs)} = 0.25e^{-0.05(0.25)} + 0.25e^{-0.05(0.5)} = 0.490722$$

So $x-y$ is

$$x - y = 40 - (30 - 0.490722) = \boxed{\mathbf{10.490722}}$$

6.26. The present value of 6 months of dividends is $3(e^{-0.06(1/8)} + e^{-0.06(3/8)}) = 5.9108$ for Sohitu and $e^{-0.005} + e^{-0.02} = 1.9752$ for Flashy. By put-call parity

$$\begin{aligned} 7.04 - 4.60 &= 180 - 5.9108 - x(90 - 1.9752) \\ 2.44 &= 174.0892 - 88.0248x \\ x &= \frac{171.6492}{88.0248} = \boxed{\mathbf{1.95}} \end{aligned}$$

6.27. Consider the option to buy x shares of A for 1 share of B as the put in put-call parity. Then the option to buy $1/x$ shares of B for 1 share of A is $1/x$ times an option to sell x shares of A for 1 share of B, which would correspond to the call in put-call parity. So

$$\begin{aligned} 2.39 - 2.74x &= 100x - 60e^{-0.03} \\ 102.74x &= 60.6176 \\ x &= \boxed{\mathbf{0.59}} \end{aligned}$$

6.28. The spot exchange rate for yen in dollars is $\frac{1}{95}$\$/¥. By put-call parity,

$$\begin{aligned} C(¥,\$,1) - P(¥,\$,1) &= x_0e^{-r_{¥}} - Ke^{-r_{\$}} \\ 0.0011 - P(¥,\$,1) &= \frac{1}{95}e^{-0.01} - 0.01e^{-0.05} \\ &= 0.0104216 - 0.0095123 = 0.0009093 \\ P(¥,\$,1) &= 0.0011 - 0.0009093 = \boxed{\mathbf{\$0.0001907}} \end{aligned}$$

6.29. The dollar-denominated put option is equivalent to a pound-denominated call option on dollars paying \$1.5 per £1. To reduce this to one paying \$1 per £(1/1.5), divide by 1.5, so the price in dollars is 0.02 and the price in pounds is £$\frac{0.02}{1.5}$ = $\boxed{\text{£}\mathbf{0.01333}}$.

6.30. We'll use the answer to the previous exercise and put-call parity.

$$\begin{aligned} C(\$,£,0.5) - P(\$,£,0.5) &= e^{-(0.06)(0.5)}x_0 - e^{-(0.03)(0.5)}K \\ 0.01333 - P(\$,£,0.5) &= 0.67e^{-0.03} - 0.67e^{-0.015} \\ 0.01333 - P(\$,£,0.5) &= -0.00983 \\ P(\$,£,0.5) &= 0.01333 + 0.00983 = \boxed{\mathbf{0.02316}} \end{aligned}$$

6.31. The put option allows giving a euro and receiving \$1.3. The call option allows receiving \$1 and giving euros. The number of euros would have to be $\frac{1}{1.3} = \boxed{\text{€}0.7692}$. The spot exchange rate isn't relevant.

6.32. By put-call parity,

$$\begin{aligned}
C(\text{€},\$,1) - P(\text{€},\$,1) &= x_0 e^{-r_{\text{€}}} - Ke^{-r_\$}\\
0.05 - 0.20 &= 1.2e^{-r_{\text{€}}} - 1.3e^{-0.05}\\
1.2e^{-r_{\text{€}}} &= -0.15 + 1.236598 = 1.086598\\
e^{-r_{\text{€}}} &= \frac{1.086598}{1.2} = 0.90550\\
r_{\text{€}} &= -\ln 0.90550 = \boxed{\mathbf{0.09927}}
\end{aligned}$$

6.33. By put-call parity,

$$\begin{aligned}
C(\text{€},¥,1) - P(\text{€},¥,1) &= x_0 e^{-r_{\text{€}}} - Ke^{-r_{¥}}\\
¥3 - ¥2 &= ¥110e^{-0.04} - Ke^{-0.02}\\
¥1 &= ¥110(0.960789) - K(0.980199)\\
K &= \frac{¥110(0.960789) - 1}{0.980199} = \boxed{\mathbf{¥106.802}}
\end{aligned}$$

6.34. By put-call parity,

$$\begin{aligned}
C(£,\$,1) - P(£,\$,1) &= x_0 e^{-0.06} - Ke^{-0.04}\\
0.05 - 0.10 &= x_0 e^{-0.06} - 1.6e^{-0.04}\\
x_0 e^{-0.06} &= -0.05 + 1.6(0.960789) = 1.48726\\
x_0 &= 1.48726e^{0.06} = 1.48726(1.061837) = \boxed{\mathbf{1.57923}}
\end{aligned}$$

6.35. If x_0 is the current (spot) exchange rate of dollars per pound, the prepaid forward exchange rate of dollars per pound is $x_0 e^{-r_£ t}$ and the forward exchange rate is $x_0 e^{(r_\$ - r_£)t}$, since you have to pay interest for time t on the prepaid forward exchange rate. So we need $x_0 e^{(0.04-0.06)(0.5)} = x_0 e^{-0.01}$.

Using put-call parity,

$$\begin{aligned}
C(£,\$,0.5) - P(£,\$,0.5) &= x_0 e^{-0.03} - Ke^{-0.02}\\
0.05 - 0.02 &= x_0 e^{-0.03} - 1.45e^{-0.02}\\
x_0 e^{-0.03} &= 0.03 + 1.45(0.980199) = 1.45129\\
x_0 e^{-0.01} &= 1.45129e^{0.02} = \boxed{\mathbf{1.4806}}
\end{aligned}$$

Quiz Solutions

6-1. Using equation (6.3),

$$\begin{aligned}
C(K,T) - P(K,T) &= S_0 - \mathrm{PV}_{0,T}(\mathrm{Divs}) - Ke^{-rT}\\
1.62 - P(K,T) &= 50 - 2e^{-0.04/3} - 50e^{-0.04(0.5)}\\
&= 50 - 2(0.9867552) - 50(0.9801987) = -0.9834\\
P(K,T) &= 1.62 + 0.9834 = \boxed{\mathbf{2.6034}}
\end{aligned}$$

6-2. Using equation (6.4),

$$\begin{aligned}C(K,T)-P(K,T)&=S_0e^{-\delta T}-Ke^{-rT}\\ C(K,T)-4.46&=57e^{-(0.03)(0.25)}-55e^{-(0.05)(0.25)}\\ &=57(0.992528)-55(0.987578)=2.2573\\ C(K,T)&=4.46+2.2573=\boxed{\mathbf{6.7173}}\end{aligned}$$

6-3. The stock price is irrelevant.
We have

$$\begin{aligned}C(K,T)-P(K,T)&=S_0e^{-\delta T}-Ke^{-rT}\\ S_0&=e^{0.01(0.5)}\big(C(45,0.5)-P(45,0.5)\big)+45e^{(-0.04+0.01)(0.5)}\end{aligned}$$

For one share of stock, the first summand indicates the puts to sell and calls to buy, and the second summand indicates the investment in Treasuries. Therefore, the amount to invest in Treasuries to synthesize 100 shares of stock is

$$100\left(45e^{(-0.04+0.01)(0.5)}\right)=4500e^{-0.015}=\boxed{\mathbf{4433}}.$$

6-4. A trick question. The option to receive 1 share of Stock A for 2 shares of Stock B is the same as the option to sell 2 shares of Stock B for 1 share of Stock A, the put option mentioned in the example, so the premium for this call option is $\boxed{\mathbf{11.50}}$.

6-5. The prepaid forward price for dollars is

$$x_0e^{-r_fT}=90e^{-0.05(0.5)}=¥87.7779$$

The prepaid forward price for the strike asset, yen, is

$$Ke^{-r_dT}=92e^{-0.01(0.5)}=¥91.5411$$

By put-call parity,

$$\begin{aligned}P(90,92,0.5)-C(90,92,0.5)&=91.5411-87.7779=3.7632\\ P(90,92,0.5)&=0.75+3.7632=\boxed{\mathbf{4.5132}}\end{aligned}$$

6-6. We divide by Kx_0 to obtain $0.0002/\big((0.005)(0.0048)\big)=\boxed{\mathbf{¥8\tfrac{1}{3}}}$.

Practice Exam 1

1. Options on XYZ stock trade on the Newark Exchange. Each option is for 100 shares. You are given that on March 21:

(i) 3000 options traded.
(ii) The price of a share of stock was \$40.
(iii) The price of each option was \$90.

Determine the total notional value of all of the options traded.

(A) 3,000 (B) 9,000 (C) 120,000 (D) 270,000 (E) 12,000,000

2. For American put options on a stock with identical expiry dates, you are given the following prices:

Strike price	Put premium
30	2.40
35	6.40

For an American put option on the same stock with the same expiry date and strike price 38, which of the following statements is correct?

(A) The lowest possible price for the option is 8.80.
(B) The highest possible price for the option is 8.80.
(C) The lowest possible price for the option is 9.20.
(D) The highest possible price for the option is 9.20.
(E) The lowest possible price for the option is 9.40.

3. A company has 100 shares of ABC stock. The current price of ABC stock is 30. ABC stock pays no dividends.

The company would like to guarantee its ability to sell the stock at the end of six months for at least 28.

European call options on the same stock expiring in 6 months with exercise price 28 are available for 4.10.

The continuously compounded risk-free interest rate is 5%.

Determine the cost of the hedge.

(A) 73 (B) 85 (C) 99 (D) 126 (E) 141

Exam questions continue on the next page . . .

4. You are given the following prices for a stock:

Time	Price
Initial	39
After 1 month	39
After 2 months	37
After 3 months	43

A portfolio of 3-month Asian options, each based on monthly averages of the stock price, consists of the following:

(i) 100 arithmetic average price call options, strike 36.
(ii) 200 geometric average strike call options.
(iii) 300 arithmetic average price put options, strike 41.

Determine the net payoff of the portfolio after 3 months.

(A) 1433 (B) 1449 (C) 1464 (D) 1500 (E) 1512

5. The price of a 6-month futures contract on widgets is 260.

A 6-month European call option on the futures contract with strike price 256 is priced using Black's formula.

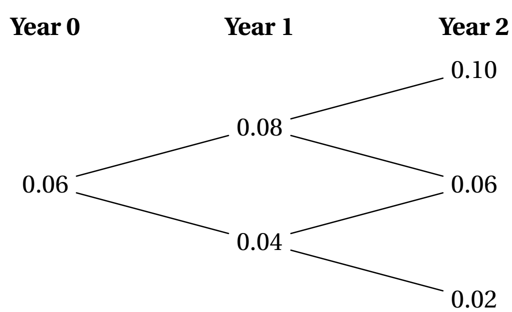
You are given:

(i) The continuously compounded risk-free rate is 0.04.
(ii) The volatility of the futures contract is 0.25.

Determine the price of the option.

(A) 19.84 (B) 20.16 (C) 20.35 (D) 20.57 (E) 20.74

6. You are given the following binomial tree for continuously compounded interest rates:



The probability of an up move is 0.5.

Calculate the continuously compounded interest rate on a default-free 3-year zero-coupon bond.

(A) 0.0593 (B) 0.0594 (C) 0.0596 (D) 0.0597 (E) 0.0598

7. Investor A bought a 40-strike European call option expiring in 1 year on a stock for 5.50. Investor A earned a profit of 6.44 at the end of the year.

Investor B bought a 45-strike European call option expiring in 1 year on the same stock at the same time, and earned a profit of 3.22 at the end of the year.

The continuously compounded risk-free interest rate is 2%.

Determine the price of the 45-strike European call option.

(A) 3.67 (B) 3.71 (C) 3.75 (D) 3.78 (E) 3.82

8. For a delta-hedged portfolio, you are given

(i) The stock price is 40.
(ii) The stock's volatility is 0.2.
(iii) The option's gamma is 0.02.

Estimate the annual variance of the portfolio if it is rehedged every half-month.

(A) 0.001 (B) 0.017 (C) 0.027 (D) 0.034 (E) 0.054

9. You own 100 shares of a stock whose current price is 42. You would like to hedge your downside exposure by buying 100 6-month European put options with a strike price of 40. You are given:

(i) The Black-Scholes framework is assumed.
(ii) The continuously compounded risk-free interest rate is 5%.
(iii) The stock pays no dividends.
(iv) The stock's volatility is 22%.

Determine the cost of the put options.

(A) 121 (B) 123 (C) 125 (D) 127 (E) 129

10. You are given the following information for a European call option expiring at the end of three years:

(i) The current price of the stock is 66.
(ii) The strike price of the option is 70.
(iii) The continuously compounded risk-free interest rate is 0.05.
(iv) The continuously compounded dividend rate of the stock is 0.02.

The option is priced using a 1-period binomial tree with $u = 1.3$, $d = 0.7$.

A replicating portfolio consists of shares of the underlying stock and a loan.

Determine the amount borrowed in the replicating portfolio.

(A) 14.94 (B) 15.87 (C) 17.36 (D) 17.53 (E) 18.43

11. You are given the following weekly stock prices for six consecutive weeks:

50.02	51.11	50.09	48.25	52.06	54.18

Estimatethe annual volatility of the stock.

(A) 0.11 (B) 0.12 (C) 0.29 (D) 0.33 (E) 0.34

Exam questions continue on the next page . . .

12. For European options on a stock having the same expiry and strike price, you are given:

(i) The stock price is 85.
(ii) The strike price is 90.
(iii) The continuously compounded risk free rate is 0.04.
(iv) The continuously compounded dividend rate on the stock is 0.02.
(v) A call option has premium 9.91.
(vi) A put option has premium 12.63.

Determine the time to expiry for the options.

(A) 3 months (B) 6 months (C) 9 months (D) 12 months (E) 15 months

13. A portfolio of European options on a stock consists of a bull spread of calls with strike prices 48 and 60 and a bear spread of puts with strike prices 48 and 60.

You are given:

(i) The options all expire in 1 year.
(ii) The current price of the stock is 50.
(iii) The stock pays dividends at a continuously compounded rate of 0.01.
(iv) The continuously compounded risk-free interest rate is 0.05.

Calculate the price of the portfolio.

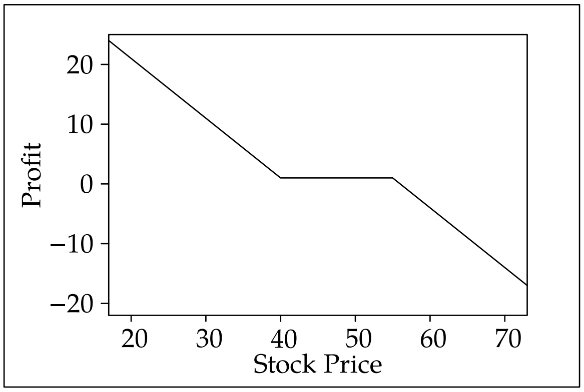
(A) 9.51 (B) 9.61 (C) 9.90 (D) 11.41 (E) 11.53

14. Which of statements (A)–(D) is *not* a weakness of the lognormal model for stock prices?

(A) Volatility is constant.
(B) Large stock movements do not occur.
(C) Projected stock prices are skewed to the right.
(D) Stock returns are not correlated over time.
(E) (A)–(D) are all weaknesses.

Exam questions continue on the next page . . .

15. You are given the following graph of the profit on a position with derivatives:



Determine which of the following positions has this profit graph.

(A) Long forward
(B) Short forward
(C) Long collar
(D) Long collared stock
(E) Short collar

16. For a put option on a stock:

(i) The premium is 2.56.
(ii) Delta is −0.62.
(iii) Gamma is 0.09.
(iv) Theta is −0.02 per day.

Calculate the delta-gamma-theta approximation for the put premium after 3 days if the stock price goes up by 2.

(A) 1.20 (B) 1.32 (C) 1.44 (D) 1.56 (E) 1.62

17. S_t is the price of a stock at time t, with t expressed in years. You are given:

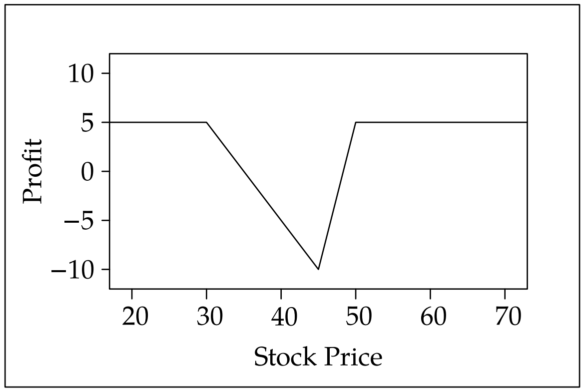
(i) S_t/S_0 is lognormally distributed.
(ii) The continuously compounded expected annual return on the stock is 5%.
(iii) The annual σ for the stock is 30%.
(iv) The stock pays no dividends.

Determine the probability that the stock will have a positive return over a period of three years.

(A) 0.49 (B) 0.51 (C) 0.54 (D) 0.59 (E) 0.61

Exam questions continue on the next page . . .

18. The following diagram is a graph of profit from an option strategy.



Determine which option strategy produces this profit graph.

(A) Long butterfly spread
(B) Short butterfly spread
(C) Ratio spread
(D) Long strangle
(E) Short strangle

19. For an at-the-money European call option on a nondividend paying stock:

(i) The price of the stock follows the Black-Scholes framework
(ii) The option expires at time t.
(iii) The option's delta is 0.5832.

Calculate delta for an at-the-money European call option on the stock expiring at time $2t$.

(A) 0.62 (B) 0.66 (C) 0.70 (D) 0.74 (E) 0.82

20. An insurance company offers a contract that pays a floating interest rate at the end of each year for 2 years. The floating rate is the 1-year-bond interest rate prevailing at the beginning of each of the two years. A rider provides that a minimum of 3% effective will be paid in each year.

You are given that the current interest rate is 5% effective for 1-year bonds and 5.5% effective for 2-year zero-coupon bonds. The volatility of a 1-year forward on a 1-year bond is 0.12.

Using the Black formula, calculate the value of the rider for an investment of 1000.

(A) 31 (B) 32 (C) 33 (D) 58 (E) 60

21. Gap options on a stock have six months to expiry, strike price 50, and trigger 49. You are given:

(i) The stock price is 45.
(ii) The continuously compounded risk free rate is 0.08.
(iii) The continuously compounded dividend rate of the stock is 0.02.

The premium for a gap call option is 1.68.

Determine the premium for a gap put option.

(A) 4.20 (B) 5.17 (C) 6.02 (D) 6.96 (E) 7.95

Exam questions continue on the next page . . .

22. Determine which of the following positions has the same cash flow as a short zero-coupon bond position.

(A) Long stock and long forward
(B) Long stock and short forward
(C) Short stock and long forward
(D) Short stock and short forward
(E) Long forward and short forward

23. A 1-year American pound-denominated put option on euros allows the sale of €100 for £90. It is modeled with a 2-period binomial tree based on forward prices. You are given

(i) The spot exchange rate is £0.8/€.
(ii) The continuously compounded risk-free rate in pounds is 0.06.
(iii) The continuously compounded risk-free rate in euros is 0.04.
(iv) The volatility of the exchange rate of pounds to euros is 0.1.

Calculate the price of the put option.

(A) 8.92 (B) 9.36 (C) 9.42 (D) 9.70 (E) 10.00

24. For a 1-year call option on a nondividend paying stock:

(i) The price of the stock follows the Black-Scholes framework.
(ii) The current stock price is 40.
(iii) The strike price is 45.
(iv) The continuously compounded risk-free interest rate is 0.05.

It has been observed that if the stock price increases 0.50, the price of the option increases 0.25.

Determine the implied volatility of the stock.

(A) 0.32 (B) 0.37 (C) 0.44 (D) 0.50 (E) 0.58

25. The price of an asset, $X(t)$, follows the Black-Scholes framework. You are given that

(i) The continuously compounded expected rate of appreciation is 0.1.
(ii) The volatility is 0.2.

Determine $\Pr\left(X(2)^3 > X(0)^3\right)$.

(A) 0.63 (B) 0.65 (C) 0.67 (D) 0.69 (E) 0.71

26. A market-maker writes a 1-year call option and delta-hedges it. You are given:

(i) The stock's current price is 100.
(ii) The stock pays no dividends.
(iii) The call option's price is 4.00.
(iv) The call delta is 0.76.
(v) The call gamma is 0.08.
(vi) The call theta is −0.02 per day.
(vii) The continuously compounded risk-free interest rate is 0.05.

The stock's price rises to 101 after 1 day.

Estimate the market-maker's profit.

(A) −0.04 (B) −0.03 (C) −0.02 (D) −0.01 (E) 0

27. You are simulating one value of a lognormal random variable with parameters $\mu = 1$, $\sigma = 0.4$ by drawing 12 uniform numbers on $[0, 1]$. The sum of the uniform numbers is 5.

Determine the generated lognormal random number.

(A) 1.7 (B) 1.8 (C) 1.9 (D) 2.0 (E) 2.1

28. Consider European put and call options on ABC Stock with expiration date 3 months from today. You are given

(i) The implied volatility of a put option whose delta is −0.25 is 0.16.
(ii) The implied volatility of a call option whose delta is 0.25 is 0.20.

Calculate the risk reversal.

(A) −0.20 (B) −0.04 (C) 0.04 (D) 0.18 (E) 0.25

29. You are given:

(i) The price of a stock is 40.
(ii) The continuous dividend rate for the stock is 0.02.
(iii) Stock volatility is 0.3.
(iv) The continuously compounded risk-free interest rate is 0.06.

A 3-month at-the-money European call option on the stock is priced with a 1-period binomial tree. The tree is constructed so that the risk-neutral probability of an up move is 0.5 and the ratio between the prices on the higher and lower nodes is $e^{2\sigma\sqrt{h}}$, where h is the amount of time between nodes in the tree.

Determine the resulting price of the option.

(A) 3.11 (B) 3.16 (C) 3.19 (D) 3.21 (E) 3.28

Exam questions continue on the next page . . .

30. For a portfolio of call options on a stock:

Number of shares of stock	Call premium per share	Delta
100	11.4719	0.6262
100	11.5016	0.6517
200	10.1147	0.9852

Calculate delta for the portfolio.

(A) 0.745 (B) 0.812 (C) 0.934 (D) 297.9 (E) 324.8

Solutions to the above questions begin on page 595.

Appendix A. Solutions for the Practice Exams

Answer Key for Practice Exam 1

1	E	11	D	21	B
2	A	12	E	22	C
3	E	13	D	23	E
4	B	14	C	24	B
5	A	15	C	25	E
6	D	16	C	26	B
7	C	17	B	27	B
8	D	18	B	28	C
9	E	19	A	29	B
10	B	20	C	30	E

Practice Exam 1

1. [**Lesson 1**] Notional value is the value of the underlying asset. Here that is 100(40) = 4000 for each option, or 4000(3000) = $\boxed{\mathbf{12{,}000{,}000}}$ for all options. **(E)**

2. [**Section 7.4**] Options are convex, meaning that as the strike price increases, the rate of increase in the put premium does not decrease. The rate of increase from 30 to 35 is $(6.40 - 2.40)/(35 - 30) = 0.80$, so the rate of increase from 35 to 38 must be at least $(38 - 35)(0.80) = 2.40$, making the price at least $6.40 + 2.40 = 8.80$. Thus **(A)** is correct.

3. [**Subsection 6.1**] By put-call parity,

$$\begin{aligned} P &= C + Ke^{-rt} - Se^{-\delta t} \\ &= 4.10 + 28e^{-0.025} - 30 = 1.4087 \end{aligned}$$

For 100 shares, the cost is 100(1.4087) = $\boxed{\mathbf{140.87}}$. **(E)**

4. [**Section 18.1**] The monthly arithmetic average of the prices is

$$\frac{39 + 37 + 43}{3} = 39.6667$$

The monthly geometric average of the prices is

$$\sqrt[3]{(39)(37)(43)} = 39.5893$$

The payments on the options are:

- The arithmetic average price call options with strike 36 pay $39.6667 - 36 = 3.6667$.

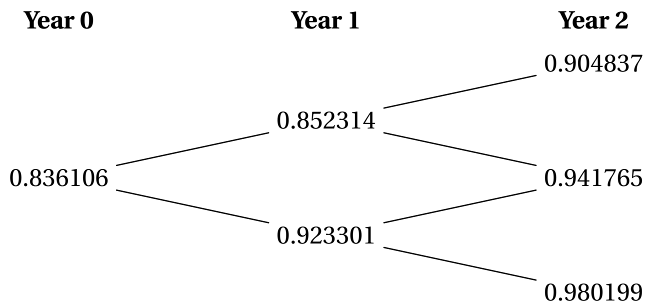


Figure A.1: Zero-coupon bond prices in the solution for question 6

- The geometric average strike call options pay $43 - 39.5893 = 3.4107$.
- The arithmetic average price put options with strike 41 pay $41 - 39.6667 = 1.3333$.

The total payment on the options is $100(3.6667) + 200(3.4107) + 300(1.3333) = \boxed{\mathbf{1448.8}}$. **(B)**

5. **[Section 14.3]** By Black's formula,

$$\begin{aligned}
d_1 &= \frac{\ln(260/256) + 0.5(0.25^2)(0.5)}{0.25\sqrt{0.5}} = 0.17609\\
d_2 &= 0.17609 - 0.25\sqrt{0.5} = -0.00068\\
N(d_1) &= N(0.17609) = 0.56989\\
N(d_2) &= N(-0.00068) = 0.49973\\
C &= 260e^{-0.02}(0.56989) - 256e^{-0.02}(0.49973) = \boxed{\mathbf{19.84}} \qquad \textbf{(A)}
\end{aligned}$$

6. **[Section 22.1]** The prices of bonds at year 2 are $e^{-0.1} = 0.904837$, $e^{-0.06} = 0.941765$, and $e^{-0.02} = 0.980199$ at the 3 nodes. Pulling back, the price of a 2-year bond at the upper node of year 1 is

$$0.5e^{-0.08}(0.904837 + 0.941765) = 0.852314$$

and the price of a 2-year bond at the lower node of year 1 is

$$0.5e^{-0.04}(0.941765 + 0.980199) = 0.923301$$

The price of a 3-year bond initially is

$$0.5e^{-0.06}(0.852314 + 0.923301) = 0.836106$$

The yield is $-(\ln 0.836106)/3 = \boxed{\mathbf{0.059667}}$. **(D)** The binomial tree of bond prices is shown in Figure A.1.

7. **[Section 4.1]** Let S be the value of the stock at the end of one year. Profit for Investor A is $S - 40 - 5.5e^{0.02} = 6.44$. Therefore $S = 52.05$.

Let C be the call price for Investor B. For Investor B, profit was $52.05 - 45 - Ce^{0.02} = 3.22$. Therefore, $C = \boxed{\mathbf{3.75}}$. **(C)**

8. **[Section 17.3]** By the Boyle-Emanuel formula, with period $\frac{1}{24}$ of a year, the variance of annual returns is

$$\mathrm{Var}(R_{1/24,1}) = \tfrac{1}{2}\big((40^2)(0.20^2)(0.02)\big)^2/24 = \boxed{\mathbf{0.0341}} \qquad \textbf{(D)}$$

9. **[Lesson 14]** For one share, Black-Scholes formula gives:

$$\begin{aligned}
d_1 &= \frac{\ln(42/40) + \bigl(0.05 - 0 + 0.5(0.22^2)\bigr)(0.5)}{0.22\sqrt{0.5}} = 0.55212 \\
d_2 &= 0.55212 - 0.22\sqrt{0.5} = 0.39656 \\
N(-d_2) &= N(-0.39656) = 0.34585 \\
N(-d_1) &= N(-0.55212) = 0.29043 \\
P &= 40e^{-0.05(0.5)}(0.34585) - 42(0.29043) = 1.2944
\end{aligned}$$

The cost of 100 puts is $100(1.2944) = \boxed{\mathbf{129.44}}$. **(E)**

Note that this question has nothing to do with delta hedging. The purchaser is merely interested in guaranteeing that he receives at least 40 for each share, and does not wish to give up upside potential. A delta hedger gives up upside potential in return for keeping loss close to zero.

10. **[Lesson 8]** $C_d = 0$ and $C_u = 1.3(66) - 70 = 15.8$. By equation (8.2),

$$B = e^{-rt}\left(\frac{uC_d - dC_u}{u - d}\right) = e^{-0.15}\left(\frac{-0.7(15.8)}{0.6}\right) = -15.87$$

$\boxed{\mathbf{15.87}}$ is borrowed. **(B)**

11. **[Lesson 13]** First calculate the logarithms of ratios of consecutive prices

t	S_t	$\ln(S_t/S_{t-1})$
0	50.02	
1	51.11	0.02156
2	50.09	−0.02016
3	48.25	−0.03743
4	52.06	0.07600
5	54.18	0.03991

Then calculate the sample standard deviation.

$$\begin{aligned}
\frac{0.02156 - 0.02016 - 0.03743 + 0.07600 + 0.03991}{5} &= 0.01598 \\
\frac{0.02156^2 + 0.02016^2 + 0.03743^2 + 0.07600^2 + 0.03991^2}{5} &= 0.001928 \\
\tfrac{5}{4}(0.001928 - 0.01598^2) &= 0.002091 \\
\sqrt{0.002091} &= 0.04573
\end{aligned}$$

Then annualize by multiplying by $\sqrt{52}$

$$0.04573\sqrt{52} = \boxed{\mathbf{0.3298}} \qquad \textbf{(D)}$$

12. **[Subsection 6.1]** By put-call parity

$$12.63 - 9.91 = 90e^{-0.04t} - 85e^{-0.02t}$$

$$90e^{-0.04t} - 85e^{-0.02t} - 2.72 = 0$$

Let $x = e^{-0.02t}$ and solve the quadratic for x.

$$x = \frac{85 + \sqrt{85^2 + 4(90)(2.72)}}{2(90)} = \frac{175.577}{180} = 0.975428$$

The other solution to the quadratic leads to $x < 0$, which is impossible for $x = e^{-0.02t}$. Now we solve for t.

$$\begin{aligned} e^{-0.02t} &= 0.975428 \\ 0.02t &= -\ln 0.975428 = 0.024879 \\ t &= 50(0.024879) = \boxed{\mathbf{1.244}} \qquad \textbf{(E)} \end{aligned}$$

13. **[Section 5.3]** This is a box spread. At expiry, the 48-strike call and put will require payment of 48, and the 60-strike call and put will result in receiving 60, so the portfolio will pay $60 - 48 = 12$. The present value of 12 is $12e^{-0.05} = \boxed{\mathbf{11.41}}$. **(D)**

14. **[Subsection 11.2.1]** **(C)** is not a weakness, since one would expect that the multiplicative change in stock price, rather than the additive change, is symmetric.

15. **[Section 5.3]** A $\boxed{\textbf{long collar}}$ has this graph. **(C)** A short forward wouldn't have the flat section. Long forwards and short collars increase in value with increasing stock prices. A collared stock has flat lines on the left and right.

16. **[Section 17.2]** Theta is expressed per day of decrease, so we just have to multiply it as given by 3. Thus the change in price is

$$\Delta\epsilon + 0.5\Gamma\epsilon^2 + \theta h = -0.62(2) + 0.5(0.09)(2^2) - 0.02(3) = -1.12$$

The new price is $2.56 - 1.12 = \boxed{\mathbf{1.44}}$. **(C)**

17. **[Section 12.2]** We are given that the average return $\alpha = 0.05$, so the parameter of the associated normal distribution is $\mu = 0.05 - 0.5(0.3^2) = 0.005$. For a three year period, $m = \mu t = 0.015$ and $v = \sigma\sqrt{t} = 0.3\sqrt{3} = 0.5196$. For a positive return, we need the normal variable with these parameters to be greater than 0. The probability that an $\mathcal{N}(0.015, 0.5196^2)$ variable is greater than 0 is $N(0.015/0.5196) = N(0.02887) = \boxed{\mathbf{0.51151}}$. **(B)**

18. **[Section 5.4]** A short butterfly spread produces this graph. **(B)** The spread is asymmetric with strike prices 30, 45, 50. A long butterfly spread would have this graph upside-down. A ratio spread would not have a flat line on both sides; neither would a strangle.

19. **[Section 15.1]** Delta is $e^{-\delta t}N(d_1)$, or $N(d_1)$ for a nondividend paying stock. Since the option is at-the-money,

$$d_1 = \frac{(r + 0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{r + 0.5\sigma^2}{\sigma}\sqrt{t}$$

So doubling time multiplies d_1 by $\sqrt{2}$.

$$N(d_1) = 0.5832$$

$$\begin{aligned}d_1 &= N^{-1}(0.5832) = 0.2101\\ d_1\sqrt{2} &= (0.2101)(1.4142) = 0.2971\\ N(0.2971) &= \boxed{\mathbf{0.6168}} \qquad \textbf{(A)}\end{aligned}$$

20. **[Section 21.3]** This is a floorlet in the second year. The value of a 1-year forward on a 1-year bond with maturity value 1 is

$$F_{0,1}\big(P(1,2)\big) = \frac{P(0,2)}{P(0,1)} = \frac{1.05}{1.055^2} = 0.943375$$

The strike price is $1/(1+K_R) = 1/1.03 = 0.970874$, and this is $1+K_R = 1.03$ calls. The Black formula gives

$$\begin{aligned}d_1 &= \frac{\ln(0.943375/0.970874) + 0.5(0.12^2)}{0.12} = -0.17944\\ d_2 &= -0.17944 - 0.12 = -0.29944\\ N(d_1) &= N(-0.17944) = 0.42880\\ N(d_2) &= N(-0.29944) = 0.38230\\ C &= \frac{1}{1.05}\big(0.943375(0.42880) - 0.970874(0.38230)\big) = 0.03176\end{aligned}$$

Multiplying by 1000, the answer is $1000\big(1.03(0.03176)\big) = \boxed{\mathbf{32.71}}$. **(C)**

21. **[Section 19.2]** For gap options, put-call parity applies with the strike price. If you buy a call and sell a put, if $S_T > K_2$ (the trigger price) you collect S_t and pay K_1, and if $S_t < K_2$ you pay K_1 and collect S_t which is the same as collecting S_t and paying K_1, so

$$C - P = Se^{-\delta t} - K_1e^{-rt}$$

In this problem,

$$P = C + K_1e^{-rt} - Se^{-\delta t} = 1.68 + 50e^{-0.04} - 45e^{-0.01} = \boxed{\mathbf{5.167}} \qquad \textbf{(B)}$$

22. **[Section 2.4]** A synthetic forward is a long stock plus a short bond. So a short bond is a long forward plus a short stock. **(C)**

23. **[Section 9.3]** The 6-month forward rate of euros in pounds is $e^{(0.06-0.04)(0.5)} = e^{0.01} = 1.01005$. Up and down movements, and the risk-neutral probability of an up movement, are

$$\begin{aligned}u &= e^{0.01+0.1\sqrt{0.5}} = 1.08406\\ d &= e^{0.01-0.1\sqrt{0.5}} = 0.94110\\ p^* &= \frac{1.01005 - 0.94110}{1.08406 - 0.94110} = 0.4823\\ 1-p^* &= 1 - 0.4823 = 0.5177\end{aligned}$$

The binomial tree is shown in Figure A.2. At the upper node of the second column, the put value is calculated as

$$P_u = e^{-0.03}(0.5177)(0.08384) = 0.04212$$

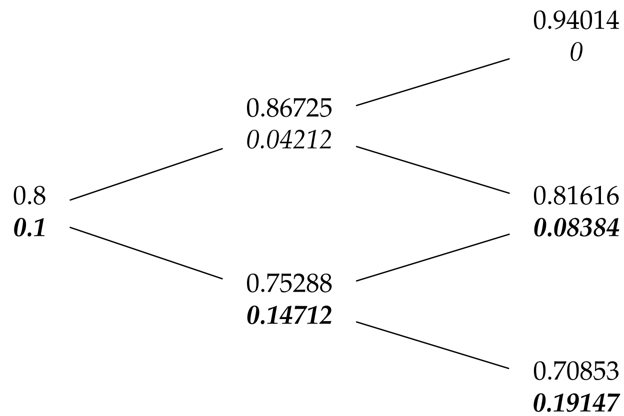


Figure A.2: Exchange rates and option values for put option of question 23

At the lower node of the second column, the put value is calculated as

$$P_d^{\text{tentative}} = e^{-0.03}\big((0.4823)(0.08384) + (0.5177)(0.19147)\big) = 0.13543$$

but the exercise value $0.9 - 0.75288 = 0.14712$ is higher so it is optimal to exercise. At the initial node, the calculated value of the option is

$$P^{\text{tentative}} = e^{-0.03}\big((0.4823)(0.04212) + (0.5177)(0.14712)\big) = 0.09363$$

Since $0.9 - 0.8 = 0.1 > 0.09363$, it is optimal to exercise the option immediately, so its value is 0.10 (which means that such an option would never exist), and the price of an option for €100 is $100(0.10) = \boxed{\mathbf{10}}$. **(E)**

24. **[Section 16.2]** Δ is observed to be $0.25/0.50 = 0.5$. In Black-Scholes formula, $\Delta = e^{-\delta t}N(d_1) = N(d_1)$ in our case. Since $N(d_1) = 0.5$, $d_1 = 0$. Then

$$\begin{aligned}
\frac{\ln(S/K) + r + 0.5\sigma^2}{\sigma} &= 0\\
\ln(40/45) + 0.05 + 0.5\sigma^2 &= 0\\
0.5\sigma^2 &= -\ln(40/45) - 0.05 = 0.11778 - 0.05 = 0.06778\\
\sigma^2 &= \frac{0.06778}{0.5} = 0.13556\\
\sigma &= \sqrt{0.13556} = \boxed{\mathbf{0.3682}} \qquad \textbf{(B)}
\end{aligned}$$

25. **[Section 12.2]** The fraction $X(2)/X(0)$ follows a lognormal distribution with parameters $m = 2\big(0.1 - 0.5(0.2^2)\big) = 0.16$ and $v = 0.2\sqrt{2}$. Cubing does not affect inequalities, so the requested probability is the same as $\Pr\big(\ln X(2) - \ln X(0) > 0\big)$, which is

$$1 - N\left(\frac{-0.16}{0.2\sqrt{2}}\right) = N(0.56569) = \boxed{\mathbf{0.7142}} \qquad \textbf{(E)}$$

26. **[Section 17.2]** By formula (17.3) with $\epsilon = 1$ and $h = 1/365$,

$$\begin{aligned}\text{Market-Maker Profit} &= -0.5\Gamma\epsilon^2 - \theta h - rh\big(S\Delta - C(S)\big)\\ &= -0.5(0.08)(1^2) + 0.02 - \frac{0.05}{365}[(100)(0.76) - 4]\\ &= -0.04 + 0.02 - 0.00986 = \boxed{-0.02986} \qquad \textbf{(B)}\end{aligned}$$

27. **[Section 20.2]** The sum of the uniform numbers has mean 6, variance 1, so we subtract 6 to standardize it.

$$5 - 6 = -1$$

We then multiply by σ and add μ to obtain a $\mathcal{N}(\mu, \sigma^2)$ random variable.

$$(-1)(0.4) + 1 = 0.6$$

Then we exponentiate.

$$e^{0.6} = \boxed{1.822} \qquad \textbf{(B)}$$

28. **[Section 16.2]** Risk reversal is the volatility of a call minus the volatility of a put. Here that is $0.20 - 0.16 = \boxed{0.04}$. **(C)**

29. **[Lesson 8]** The risk-neutral probability is

$$0.5 = p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{(0.06-0.02)(0.25)} - d}{u - d} = \frac{e^{0.01} - d}{u - d}$$

but $u = de^{2\sigma\sqrt{h}} = de^{2(0.3)(1/2)} = de^{0.3}$, so

$$\begin{aligned}e^{0.01} - d &= 0.5\left(e^{0.3}d - d\right)\\ e^{0.01} &= d\left(0.5(e^{0.3} - 1) + 1\right) = 1.17493d\\ d &= \frac{e^{0.01}}{1.17493} = 0.85967\\ u &= 0.85967e^{0.3} = 1.16043\end{aligned}$$

The option only pays at the upper node. The price of the option is

$$C = e^{-rh}p^*(Su - K) = e^{-0.06(0.25)}(0.5)\big(40(1.16043) - 40\big) = \boxed{3.1609} \qquad \textbf{(B)}$$

30. **[Subsection 15.1.7]** Delta for a portfolio of options on a single stock is the sum of the individual deltas of the options.

$$100(0.6262) + 100(0.6517) + 200(0.9852) = \boxed{324.8} \qquad \textbf{(E)}$$

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