Practice Exam 1

1. Losses for an insurance coverage have the following cumulative distribution function:

$F(0) = 0$
$F(1{,}000) = 0.2$
$F(5{,}000) = 0.4$
$F(10{,}000) = 0.9$
$F(100{,}000) = 1$

with linear interpolation between these values.

Calculate the hazard rate at 9,000, $h(9{,}000)$.

(A) 0.0001 (B) 0.0004 (C) 0.0005 (D) 0.0007 (E) 0.0010

2. You are given the following data on loss sizes:

Loss Amount	Number of Losses
0– 1000	5
1000– 5000	4
5000–10000	3

An ogive is used as a model for loss sizes.

Determine the fitted median.

(A) 2000 (B) 2200 (C) 2500 (D) 3000 (E) 3083

3. In a mortality study on 5 individuals, death times were originally thought to be 1, 2, 3, 4, 5. It then turned out that one of these five observations was a censored observation rather than an actual death.

Determine the observation time of the five for which turning it into a censored observation would result in the lowest variance of the Nelson-Åalen estimator of $H(4)$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. For an insurance coverage, the number of claims per year follows a Poisson distribution. Claim size follows a Pareto distribution with $\alpha = 3$. Claim counts and claim sizes are independent.

The methods of limited fluctuation credibility are used to determine premiums. The standard for full credibility is that actual aggregate claims be within 5% of expected aggregate claims 95% of the time. Based on this standard, 10,000 exposure units are needed for full credibility, where an exposure unit is a year of experience for a single insured.

Determine the expected number of claims per year.

(A) Less than 0.45
(B) At least 0.45, but less than 0.50
(C) At least 0.50, but less than 0.55
(D) At least 0.55, but less than 0.60
(E) At least 0.60

Exercises continue on the next page . . .

5. Which of the following statements is true?

(A) If data grouped into 7 groups are fitted to an inverse Pareto, the chi-square test of goodness of fit will have 5 degrees of freedom.

(B) The Kolmogorov-Smirnov statistic may be used to test the fit of a discrete distribution.

(C) The critical values of the Kolmogorov-Smirnov statistic do not require adjustment for estimated parameters.

(D) The critical values of the Kolmogorov-Smirnov statistic do not vary with sample size.

(E) The critical values of the Anderson-Darling statistic do not vary with sample size.

6–7. *Use the following information for questions 6 and 7:*

> There are two classes of insureds. In Class A, the number of claims per year has a Poisson distribution with mean 0.1 and claim size has an exponential distribution with mean 500. In Class B, the number of claims per year has a Poisson distribution with mean 0.2 and claim size has an exponential distribution with mean 250. Each class has the same number of insureds.
>
> An insured selected at random submits two claims in one year. Claim sizes are 200 and 400.

6. Calculate the probability that the insured is in class A.

(A) 0.01 (B) 0.04 (C) 0.15 (D) 0.19 (E) 0.27

7. Calculate the Bühlmann estimate of the aggregate losses for this insured in the following year.

(A) Less than 42

(B) At least 42, but less than 47

(C) At least 47, but less than 52

(D) At least 52, but less than 57

(E) At least 57

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8. A class takes an exam. Half the students are good and half the students are bad. For good students, grades are distributed according to the probability density function

$$f(x)=\frac{4}{100}\left(\frac{x}{100}\right)^3 \qquad 0\le x\le 100$$

For bad students, grades are distributed according to the probability density function

$$f(x)=\left(\frac{2}{100}\right)\left(\frac{x}{100}\right) \qquad 0\le x\le 100$$

The passing grade is 65.

Determine the average grade on this exam for a passing student.

(A) 84.8 (B) 84.9 (C) 85.0 (D) 85.1 (E) 85.2

Exercises continue on the next page . . .

9. In a mortality study on five lives, death times were 6, 7, 9, 15, and 30. Using the empirical distribution, $S(10)$ is estimated as 0.4.

To approximate the mean square error of the estimate, the bootstrap method is used. Five bootstrap samples are:

1. 6, 7, 7, 9, 30
2. 9, 6, 30, 7, 9
3. 30, 6, 6, 15, 7
4. 6, 15, 7, 9, 9
5. 30, 9, 6, 15, 15

Calculate the bootstrap approximation of the mean square error of the estimate.

(A) 0.032 (B) 0.034 (C) 0.036 (D) 0.038 (E) 0.040

10. An insurance coverage covers two types of insureds, A and B. There are an equal number of insureds in each class. Claim sizes in each class follow a Pareto distribution. Number of claims and claim sizes for insureds in each class have the following distributions:

Number of claims

	A	B
0	0.9	0.8
1	0.1	0.2

Size of claims (Pareto parameters)

	A	B
α	3	3
θ	50	60

Within each class, claim size and number of claims are independent.

Calculate the Bühlmann credibility to assign to 2 years of data.

(A) 0.01 (B) 0.02 (C) 0.03 (D) 0.04 (E) 0.05

11. An auto collision coverage is sold with deductibles of 500 and 1000. You have the following information for total loss size (including the deductible) on 86 claims:

Deductible 1000

Loss size	Number of losses
1000–2000	20
Over 2000	10

Deductible 500

Loss size	Number of losses
500–1000	32
Over 1000	24

Ground up underlying losses for both deductibles are assumed to follow an exponential distribution with the same parameter. You estimate the parameter using maximum likelihood.

Determine the fitted average total loss size (including the deductible) for claim payments on policies with a 500 deductible.

(A) 671 (B) 707 (C) 935 (D) 1171 (E) 1207

Exercises continue on the next page . . .

12. You are given the following data from a 2-year mortality study.

Duration j	Entries d_j	Withdrawals u_j	Deaths x_j
0	1000	100	33
1	500	100	c

Withdrawals and new entries are assumed to occur uniformly.

Using a Kaplan-Meier type approximation for large data sets on this data, q_1 is estimated as 0.03.

Determine c.

(A) 26 (B) 32 (C) 35 (D) 38 (E) 41

13. The number of claims per year on an insurance coverage has a binomial distribution with parameters $m=2$ and Q. Q varies by insured and is distributed according to the following density function:

$$f(q)=42q(1-q)^5 \qquad 0\le q\le 1$$

An insured submits 1 claim in 4 years.

Calculate the posterior probability that for this insured, q is less than 0.25.

(A) 0.52 (B) 0.65 (C) 0.70 (D) 0.76 (E) 0.78

14. You simulate a random variable with probability density function

$$f(x)=\begin{cases}-2x & -1\le x\le 0\\ 0 & \text{otherwise}\end{cases}$$

using the inversion method.

You use the following numbers random numbers from the uniform distribution on $[0,1]$:

0.2 0.4 0.3 0.7

Calculate the mean of the simulated observations.

(A) −0.7634 (B) −0.6160 (C) −0.2000 (D) 0.6160 (E) 0.7634

15. You are given a sample of 5 claims:

$$2,\ 3,\ 4,\ x_1,\ x_2$$

with $x_2>x_1$. This sample is fitted to a Pareto distribution using the method of moments. The resulting parameter estimates are $\hat\alpha=47.71$, $\hat\theta=373.71$.

Determine x_1.

(A) 6.0 (B) 6.6 (C) 7.0 (D) 7.6 (E) 8.0

Exercises continue on the next page . . .

16. The number of claims per year on a policy follows a Poisson distribution with parameter Λ. Λ has a uniform distribution on $(0,2)$.

An insured submits 5 claims in one year.

Calculate the Bühlmann credibility estimate of the number of claims for the following year.

(A) 1.6 (B) 1.8 (C) 2.0 (D) 2.5 (E) 3.5

17. A group has 100 lives. For each individual in this group, the mortality rate $q_x = 0.01$. Mortality for each individual is independent.

You are to simulate 3 years of mortality experience for this group using the inversion method. Use the following 3 numbers from the uniform distribution on $[0,1]$: 0.12, 0.35, 0.68.

Calculate the total number of simulated deaths over 3 years.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

18. A study is performed on the amount of time on unemployment. The records of 10 individuals are examined. 7 of the individuals are not on unemployment at the time of the study. The following is the number of weeks they were on unemployment:

5, 8, 10, 11, 17, 20, 26

3 individuals are still on unemployment at the time of the study. They have been unemployed for the following number of weeks:

5, 20, 26

Let T be the amount of time on unemployment.

Using the Kaplan-Meier estimator with exponential extrapolation past the last study time, estimate $\Pr(20 \le T \le 30)$.

(A) 0.17 (B) 0.21 (C) 0.28 (D) 0.32 (E) 0.43

19. Annual claim counts per risk are binomial with parameters $m = 2$ and Q. Q varies by risk uniformly on $(0.25, 0.75)$.

For a risk selected at random, determine the probability of no claims.

(A) 0.14 (B) 0.25 (C) 0.26 (D) 0.27 (E) 0.28

Exercises continue on the next page ...

20. The distribution of auto insurance policyholders by number of claims submitted in the last year is as follows:

Number of claims	Number of insureds
0	70
1	22
2	6
3	2
Total	100

The number of claims for each insured is assumed to follow a Poisson distribution.

Use semi-parametric empirical Bayes estimation methods, with unbiased estimators for the variance of the hypothetical mean and the expected value of the process variance, to calculate the expected number of claims in the next year for a policyholder with 2 claims in the last year.

(A) Less than 0.52
(B) At least 0.52, but less than 0.57
(C) At least 0.57, but less than 0.62
(D) At least 0.62, but less than 0.67
(E) At least 0.67

21. X is a random variable. Simulation is used to estimate $F_X(500)$. Fifty psuedorandom values are generated. Of these fifty values, twenty values are less than or equal to 500.

Estimate the number of psuedorandom values that need to be generated in order to have 95% confidence that the estimate of $F_X(500)$ is within 5% of the true value.

(A) Less than 1600
(B) At least 1600, but less than 1800
(C) At least 1800, but less than 2000
(D) At least 2000, but less than 2200
(E) At least 2200

22. For an insurance, the number of claims per year for each risk has a Poisson distribution with mean Λ. Λ varies by risk according to a gamma distribution with mean 0.5 and variance 1. Claim sizes follow a Weibull distribution with $\theta = 5$, $\tau = \frac{1}{2}$. Claim sizes are independent of each other and of claim counts.

Determine the variance of aggregate claims.

(A) 256.25 (B) 312.50 (C) 350.00 (D) 400.00 (E) 450.00

23. You are given the following claims data from an insurance coverage with claims limit 10,000:

1000, 2000, 2000, 2000, 4000, 5000, 5000

There are 3 claims for amounts over 10,000 which are censored at 10,000.

You fit this experience to an exponential distribution with parameter $\theta = 6{,}000$.

Calculate the Kolmogorov-Smirnov statistic for this fit.

(A) 0.11 (B) 0.13 (C) 0.15 (D) 0.18 (E) 0.19

Exercises continue on the next page . . .

24. For an insurance coverage, you observe the following claims sizes:

400, 1100, 1100, 3000, 8000

You fit the loss distribution to a lognormal with $\mu = 7$ using maximum likelihood.

Determine the mean of the fitted distribution.

(A) Less than 2000
(B) At least 2000, but less than 2500
(C) At least 2500, but less than 3000
(D) At least 3000, but less than 3500
(E) At least 3500

25. In a mortality study performed on 5 lives, ages at death were

70, 72, 74, 75, 75

Estimate $S(75)$ using kernel smoothing with a uniform kernel with bandwidth 4.

(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.5 (E) 0.6

26. For an insurance coverage, the number of claims per year follows a Poisson distribution with mean θ. The size of each claim follows an exponential distribution with mean 1000θ. Claim count and size are independent given θ.

You are examining one year of experience for four randomly selected policyholders, whose claims are as follows:

Policyholder #1	2000, 4000, 4000, 7000
Policyholder #2	4000
Policyholder #3	2000, 3000
Policyholder #4	1000, 4000, 5000

You use maximum likelihood to estimate θ.

Determine the variance of aggregate losses based on the fitted distribution.

(A) Less than 48,000,000
(B) At least 48,000,000, but less than 50,000,000
(C) At least 50,000,000, but less than 52,000,000
(D) At least 52,000,000, but less than 54,000,000
(E) At least 54,000,000

27. You are given:

(i) The annual number of claims for each risk follows a Poisson distribution with parameter Λ.
(ii) Λ varies by insured according to a gamma distribution with $\alpha = 3$ and $\theta = 0.1$.
(iii) Claims sizes follows a Pareto distribution with $\alpha = 3$ and $\theta = 20{,}000$.
(iv) Claim sizes are independent of claim counts.
(v) Your department handles only claims with sizes below 10,000.

Determine the variance of the annual number of claims handled per risk in your department.

(A) 0.159 (B) 0.163 (C) 0.226 (D) 0.232 (E) 0.330

Exercises continue on the next page . . .

28. Past data on aggregate losses for two group policyholders is given in the following table.

Group		Year 1	Year 2
A	Total losses	1000	1200
	Number of members	40	50
B	Total losses	500	600
	Number of members	20	40

Calculate the credibility factor used for Group A's experience using non-parametric empirical Bayes estimation methods.

(A) Less than 0.40
(B) At least 0.40, but less than 0.45
(C) At least 0.45, but less than 0.50
(D) At least 0.50, but less than 0.55
(E) At least 0.55

29. For an insurance coverage, claim size follows a Pareto distribution with parameters $\alpha = 4$ and θ. θ varies by insured and follows a normal distribution with $\mu = 3$ and $\sigma = 1$.

Determine the Bühlmann credibility to be assigned to a single claim.

(A) 0.05 (B) 0.07 (C) 0.10 (D) 0.14 (E) 0.20

30. You are given the following information regarding loss sizes:

d	Mean excess loss $e(d)$	$F(d)$
0	3000	0.0
500	2800	0.1
10,000	2600	0.8

Determine the average payment per loss for a policy with a deductible of 500 and a maximum covered loss of 10,000.

(A) Less than 1600
(B) At least 1600, but less than 1800
(C) At least 1800, but less than 2000
(D) At least 2000, but less than 2200
(E) At least 2200

Exercises continue on the next page . . .

31. A claims adjustment facility adjusts all claims for amounts less than or equal to 10,000. Claims for amounts greater than 10,000 are handled elsewhere.

In 2002, the claims handled by this facility fell into the following ranges:

Size of Claim	Number of Claims
Less than 1000	100
1000– 5000	75
5000–10000	25

The claims are fitted to a parametric distribution using maximum likelihood.

Which of the following is the correct form for the likelihood function of this experience?

(A) $\big(F(1000)\big)^{100}\big(F(5000)-F(1000)\big)^{75}\big(F(10{,}000)-F(5000)\big)^{25}$

(B) $\dfrac{\big(F(1000)\big)^{100}\big(F(5000)-F(1000)\big)^{75}\big(F(10{,}000)-F(5000)\big)^{25}}{\big(F(10{,}000)\big)^{200}}$

(C) $\dfrac{\big(F(1000)\big)^{100}\big(F(5000)-F(1000)\big)^{75}\big(F(10{,}000)-F(5000)\big)^{25}}{\big(1-F(10{,}000)\big)^{200}}$

(D) $\dfrac{\big(1-F(1000)\big)^{100}\big(F(5000)-F(1000)\big)^{75}\big(F(10{,}000)-F(5000)\big)^{25}}{\big(F(10{,}000)\big)^{200}}$

(E) $\dfrac{\big(1-F(1000)\big)^{100}\big(F(5000)-F(1000)\big)^{75}\big(F(10{,}000)-F(5000)\big)^{25}}{\big(1-F(10{,}000)\big)^{200}}$

32. For a sample from an exponential distribution, which of the following statements is false?

(A) If the sample has size 2, the sample median is an unbiased estimator of the population median.

(B) If the sample has size 2, the sample median is an unbiased estimator of the population mean.

(C) If the sample has size 3, the sample mean is an unbiased estimator of the population mean.

(D) If the sample has size 3, 1.2 times the sample median is an unbiased estimator of the population mean.

(E) The sample mean is a consistent estimator of the population mean.

33. The median of a sample is 5. The sample is fitted to a mixture of two exponential distributions with means 3 and $x > 3$, using percentile matching to determine the weights to assign to each exponential.

Which of the following is the range of values for x for which percentile matching works?

(A) $3 < x < 4.3281$.

(B) $3 < x < 7.2135$.

(C) $4.3281 < x < 7.2135$.

(D) $x > 4.3281$.

(E) $x > 7.2135$.

Exercises continue on the next page . . .

34. A random variable X has the probability density function

$$f(x)=\frac{32e^{-4/x}}{x^4}$$

$\bar{X}$ is the sample mean of 100 observations of X.

Using the normal approximation, estimate $\Pr(\bar{X}<2.5)$.

(A) 0.6915 (B) 0.8413 (C) 0.9332 (D) 0.9772 (E) 0.9938

35. Losses follow a lognormal distribution with $\mu=3$, $\sigma=0.5$.

Calculate the Value at Risk measure at security level $p=95\%$.

(A) Less than 40

(B) At least 40, but less than 45

(C) At least 45, but less than 50

(D) At least 50, but less than 55

(E) At least 55

Solutions to the above questions begin on page 1167.

Appendix A. Solutions to the Practice Exams

Answer Key for Practice Exam 1

1	C	11	E	21	E	31	B
2	A	12	B	22	D	32	A
3	D	13	D	23	D	33	E
4	E	14	A	24	A	34	E
5	E	15	C	25	B	35	C
6	D	16	C	26	E		
7	C	17	B	27	C		
8	D	18	D	28	E		
9	A	19	D	29	A		
10	A	20	E	30	D		

Practice Exam 1

1. [**Lesson 1**] $F(9{,}000) = 0.4 + \left(\frac{9{,}000-5{,}000}{10{,}000-5{,}000}\right)(0.9-0.4) = 0.8$. Then $S(9{,}000) = 0.2$. The hazard rate is $\frac{f(x)}{S(x)}$. The density function is the derivative of F, which at 9,000 is the slope of the line from 5,000 to 10,000, which is $\frac{0.9-0.4}{10{,}000-5{,}000} = 0.0001$. The answer is

$$h(9{,}000) = \frac{f(9{,}000)}{S(9{,}000)} = \frac{0.0001}{0.2} = \boxed{\mathbf{0.0005}} \qquad \textbf{(C)}$$

2. [**Lesson 20**] The distribution function at 1000, $F(1000)$, is $\frac{5}{12}$, and $F(5000) = \frac{9}{12}$. By definition, the median is the point m such that $F(m) = 0.5$. The ogive linearly interpolates between 1000 and 5000. Thus we solve the equation

$$\begin{aligned} \frac{m-1000}{5000-1000} &= \frac{0.5-\frac{5}{12}}{\frac{9}{12}-\frac{5}{12}} \\ m-1000 &= \frac{1}{4}(4000) = 1000 \\ m &= \boxed{\mathbf{2000}} \qquad \textbf{(A)} \end{aligned}$$

3. [**Lesson 24**] The variance is $\sum \frac{1}{r_i^2}$. To minimize the variance, maximize the r_i's. The r_i's for the first 3 deaths are 3 out of {5,4,3,2}. By making $\boxed{\mathbf{4}}$ the censored observation, the r_i's are 5, 4, and 3. (Making 5 the censored observation leads to a sum of *4* terms instead of 3, with the first 3 terms being the same as if 4 is censored, so it does not lead to minimal variance). **(D)**

4. [**Lesson 38**] The credibility formula in terms of expected number of claims requires $1+CV_s^2$, or the second moment divided by the first moment squared of the severity distribution. For a Pareto with $\alpha = 3$, this is

$$\frac{\frac{2\theta^2}{(2)(1)}}{\left(\frac{\theta}{2}\right)^2} = 4$$

The expected number of claims needed for full credibility is then $\left(\frac{1.96}{0.05}\right)^2(4) = 6146.56$, so we have

$$6146.56 = 10{,}000\lambda$$
$$\lambda = \boxed{\mathbf{0.614656}} \qquad \textbf{(E)}$$

5. [**Lessons 34, 35, 36**] (A) is false because the number of degrees of freedom is $n-1$ minus the number of parameters estimated. Here $n=7$ and the inverse Pareto has 2 parameters, so there are 4 degrees of freedom.

(B) is false, as indicated on page 626.

(C) is false, as indicated on page 625, where it says that the indicated critical values only work when the distribution is completely specified, not when parameters have been estimated.

(D) is false; in fact, the critical values get divided by $\sqrt{n}$.

(E) is true.

6. [**Lesson 41**] The likelihood that an insured in class A submits 2 claims for 200 and 400 is the product of the Poisson probability of 2 claims and the 2 densities under the exponential distribution with mean 500, or

$$f(200,400 \mid A) = e^{-0.1}\left(\frac{0.1^2}{2!}\right)\left(\frac{e^{-200/500}}{500}\right)\left(\frac{e^{-400/500}}{500}\right) = 5.4506 \times 10^{-9}$$

and similarly for an insured in class B:

$$f(200,400 \mid B) = e^{-0.2}\left(\frac{0.2^2}{2!}\right)\left(\frac{e^{-200/250}}{250}\right)\left(\frac{e^{-400/250}}{250}\right) = 2.3768 \times 10^{-8}$$

Since the classes are of equal size, these are the relative probabilities of the two classes. Then the probability that the insured is in class A is

$$\Pr(A \mid 200,400) = \frac{5.4506 \times 10^{-9}}{5.4506 \times 10^{-9} + 2.3768 \times 10^{-8}} = \boxed{\mathbf{0.1865}} \qquad \textbf{(D)}$$

7. [**Lesson 48**] Since the hypothetical means are the same in both classes—(0.1)(500) = 50 and (0.2)(250) = 50—$a = 0$ and there is no credibility. Thus, $\boxed{\mathbf{50}}$ is the credibility premium. **(C)**

8. [**Lesson 41**] We need to calculate $\mathbf{E}[X \mid X > 65]$, where X is the grade. By definition

$$\mathbf{E}[X \mid X > 65] = \frac{\int_{65}^{100} x f(x)\,\mathrm{d}x}{1 - F(65)}$$

$f(x)$ is an equally weighted mixture of the good and bad students, and therefore is

$$f(x) = \frac{1}{2}\left(\frac{4}{100}\left(\frac{x}{100}\right)^3 + \frac{2}{100}\left(\frac{x}{100}\right)\right)$$

First we calculate the denominator of $\mathbf{E}[X \mid X > 65]$.

$$F(x) = \int_0^x f(u)\,\mathrm{d}u = 0.5\left(\left(\frac{x}{100}\right)^4 + \left(\frac{x}{100}\right)^2\right)$$
$$F(65) = 0.5(0.65^4 + 0.65^2) = 0.300503$$
$$1 - F(65) = 1 - 0.300503 = 0.699497$$

Then we calculate the numerator.

$$\begin{aligned}\int_{65}^{100} x f(x)\mathrm{d}x &= \int_{65}^{100} 0.5\left(\frac{4x^4}{100^4}+\frac{2x^2}{100^2}\right)\mathrm{d}x\\ &= 0.5\left(\frac{4x^5}{5(100^4)}+\frac{2x^3}{3(100^2)}\right)_{65}^{100}\\ &= 0.5\left(\frac{400}{5}-\frac{4(65^5)}{5(100^4)}+\frac{200}{3}-\frac{2(65^3)}{3(100^2)}\right)\\ &= 0.5(80-9.2823+66.6667-18.3083)=59.5380\\ \mathbf{E}[X\mid X>65] &= \frac{59.5380}{0.699497}=\boxed{\mathbf{85.1155}} \qquad \textbf{(D)}\end{aligned}$$

An alternative way to solve this problem is to use the tabular form for Bayesian credibility that we studied in Lesson 41. The table would look like this:

	Good Students	Bad Students	
Prior probabilities	0.50	0.50	
Likelihood of experience	0.821494	0.5775	
Joint probabilities	0.410747	0.28875	0.699497
Posterior probabilities (p_i)	0.587203	0.412797	
65+$e(65)$	86.084252	83.737374	
$[65+e(65)]\times p_i$	50.548957	34.566511	**85.1155**

The second line, the likelihood, is derived as follows:

$$F(65)=\int_0^{65} f(x)\,\mathrm{d}x=\begin{cases}(65/100)^4 & \text{for good students}\\ (65/100)^2 & \text{for bad students}\end{cases}$$

so the likelihood of more than 65 is $1-0.65^4=0.821494$ for good students and $1-0.65^2=0.5775$ for bad students.

The fifth line, the average grade of those with grades over 65, is derived as follows for good students:

$$\begin{aligned}65+e(65) &= 65+\frac{\int_{65}^{100} S(x)\,\mathrm{d}x}{S(65)}\\ &= 65+\int_{65}^{100}\left(1-\left(\frac{x}{100}\right)^4\right)\mathrm{d}x\\ &= 65+35-\left(\frac{x^5}{5(100^4)}\right)\Bigg|_{65}^{100}\\ &= 100-\frac{100^5-65^5}{5(100^4)}=86.084252\end{aligned}$$

and for bad students, replace the 4's with 2's and the 5's with 3's to obtain $100-\frac{100^3-65^3}{2(100^2)}=83.737374$.

9. [**Lesson 58**] The estimates of $S(10)$ from these 5 samples are the proportion of numbers above 10, which are 0.2, 0.2, 0.4, 0.2, 0.6 respectively, so the bootstrap approximation is

$$\frac{(0.2-0.4)^2+(0.2-0.4)^2+(0.4-0.4)^2+(0.2-0.4)^2+(0.6-0.4)^2}{5}=\boxed{\mathbf{0.032}} \qquad \textbf{(A)}$$

10. [**Lesson 48**] Let μ_A be the hypothetical mean for A, v_A the process variance for A, and use the same notation with subscripts B for B. For process variance, we will use the compound variance formula to compute the process variances. In the case of Class A (with N and X being frequency and severity respectively):

$$\mathbf{E}[N] = 0.1 \qquad \mathrm{Var}(N) = 0.1(0.9) = 0.09$$

$$\mathbf{E}[X] = \frac{\theta}{\alpha-1} = \frac{50}{2} = 25 \qquad \mathbf{E}[X^2] = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} = \frac{2(50^2)}{(2)(1)} = 2500$$

so $\mathrm{Var}(X) = 2500 - 625$. A similar calculation is done for Class B. So using the compound variance formula for v_i, we have

$$\mu_A = 0.1(25) = 2.5 \qquad v_A = 0.1(2500-625) + 0.09(25^2) = 243.75$$

$$\mu_B = 0.2(30) = 6 \qquad v_B = 0.2(3600-900) + 0.16(30^2) = 684$$

$$a = (6-2.5)^2\left(\frac{1}{4}\right) = 3.0625 \qquad v = \frac{1}{2}(243.75+684) = 463.875$$

For 2 years of experience, the credibility is

$$Z = \frac{na}{na+v} = \frac{(2)(3.0625)}{(2)(3.0625)+463.875} = \boxed{\mathbf{0.013032}} \qquad \textbf{(A)}$$

11. [**Lesson 29**] The likelihood function (either using the fact that the exponential is memoryless, or else writing it all out and canceling out the denominators) is

$$L(\theta) = \left(1-e^{-(1000/\theta)}\right)^{20}\left(e^{-(1000/\theta)}\right)^{10}\left(1-e^{-(500/\theta)}\right)^{32}\left(e^{-(500/\theta)}\right)^{24}$$

Logging and differentiating:

$$l(\theta) = 20\ln\left(1-e^{-1000/\theta}\right) + 32\ln\left(1-e^{-500/\theta}\right) - \frac{10{,}000+12{,}000}{\theta}$$

$$\frac{\mathrm{d}l}{\mathrm{d}\theta} = \frac{-20{,}000e^{-1000/\theta}}{\theta^2(1-e^{-1000/\theta})} + \frac{-16{,}000e^{-500/\theta}}{\theta^2(1-e^{-500/\theta})} + \frac{22{,}000}{\theta^2} = 0$$

Multiply through by $\frac{\theta^2}{1000}$, and set $x = e^{-500/\theta}$. We obtain

$$22 - \frac{20x^2}{1-x^2} - \frac{16x}{1-x} = 0$$

$$\frac{22(1-x^2) - 20x^2 - 16x(1+x)}{1-x^2} = 0$$

$$22 - 22x^2 - 20x^2 - 16x - 16x^2 = 0$$

$$58x^2 + 16x - 22 = 0$$

$$x = \frac{-16+\sqrt{16^2+4(22)(58)}}{116} = 0.493207$$

$$\theta = \frac{-500}{\ln x} = 707.3875$$

An easier way to do this would be to make the substitution $x = e^{-500/\theta}$ right in $L(\theta)$, and to immediately recognize that $1-x^2 = (1-x)(1+x)$. This would avoid the confusing differentiation step:

$$l(\theta) = 20\ln(1-x^2) + 44\ln x + 32\ln(1-x)$$

$$\begin{aligned}
&= 20\ln(1-x)+20\ln(1+x)+44\ln x+32\ln(1-x)\\
&= 52\ln(1-x)+20\ln(1+x)+44\ln x\\
\frac{\mathrm{d}l}{\mathrm{d}\theta} &= -\frac{52}{1-x}+\frac{20}{1+x}+\frac{44}{x}=0\\
&-52(x)(1+x)+20(x)(1-x)+44(1-x)^2=0\\
&-52x^2-52x-20x^2+20x+44-44x^2=0\\
&116x^2+32x-44=0
\end{aligned}$$

which is double the quadratic above, and leads to the same solution for θ.

Using the fact that the exponential distribution is memoryless, the average total loss size for a 500 deductible is $500+707.3875=$ **1207.3875**. **(E)**

12. [**Lesson 26**] The conditional probability of death at the second duration (note that 0 is the first duration and 1 is the second), q_1, is estimated by $\frac{s_2}{r_2}$, number of deaths over the risk set in duration 2. Duration 2 starts with $1000-100-33=867$ lives, and since withdrawals and new entries occur uniformly, we add half the new entries and subtract half the withdrawals to arrive at $r_2=867+0.5(500-100)=1067$. Then

$$\begin{aligned}
0.03=\hat{q}_1&=\frac{c}{1067}\\
c&=\boxed{\mathbf{32}} \qquad \textbf{(B)}
\end{aligned}$$

13. [**Lesson 45**] The prior distribution is a beta distribution with $a=2$, $b=6$. (In general, in a beta distribution, a is 1 more than the exponent of q and b is 1 more than the exponent of $1-q$.) The number of claims is binomial, which means that 2 claims are possible each year. Of the 8 possible claims in 4 years, you received 1 and didn't receive 7. Thus $a'=2+1=3$ and $b'=6+7=13$ are the parameters of the posterior beta. The density function for the posterior beta is

$$\begin{aligned}
\pi(q|\mathbf{x})&=\frac{\Gamma(3+13)}{\Gamma(3)\Gamma(13)}q^2(1-q)^{12}\\
&=1365q^2(1-q)^{12}
\end{aligned}$$

since $\frac{15!}{2!12!}=\frac{(15)(14)(13)}{2}=1365$. We must integrate this function from 0 to 0.25 to obtain $\Pr(Q<0.25)$. It is easier to integrate if we change the variable, by setting $q'=1-q$. Then we have

$$\begin{aligned}
\Pr(Q<0.25)&=1365\int_{0.75}^{1}(1-q')^2q'^{12}\mathrm{d}q'\\
&=1365\int_{0.75}^{1}(q'^{12}-2q'^{13}+q'^{14})\mathrm{d}q'\\
&=\left.\frac{q'^{13}}{13}-\frac{2q'^{14}}{14}+\frac{q'^{15}}{15}\right|_{0.75}^{1}\\
&=1365(0.0007326-0.0001730)=\boxed{\mathbf{0.7639}} \qquad \textbf{(D)}
\end{aligned}$$

14. [**Lesson 55**] The distribution function is

$$F(x)=\int_{-1}^{x}-2u\,\mathrm{d}u=-u^2\Big|_{-1}^{x}=1-x^2$$

Inverting,

$$\begin{aligned} u &= 1 - x^2 \\ 1 - u &= x^2 \\ x &= -\sqrt{1-u} \end{aligned}$$

It is necessary to use the negative square root, since the simulated observation must be between −1 and 0. So

$$\begin{aligned} x_1 &= -\sqrt{1-0.2} = -0.8944 \\ x_2 &= -\sqrt{1-0.4} = -0.7746 \\ x_3 &= -\sqrt{1-0.3} = -0.8367 \\ x_4 &= -\sqrt{1-0.7} = -0.5477 \end{aligned}$$

$$\frac{-0.8944-0.7746-0.8367-0.5477}{4} = \boxed{\mathbf{-0.7634}} \qquad \textbf{(A)}$$

15. [**Lesson 27**] We write the moment equations for the first and second moments:

$$\begin{aligned} \frac{2+3+4+x_1+x_2}{5} &= \frac{\theta}{\alpha-1} \\ 9+x_1+x_2 &= 5\left(\frac{373.71}{47.71-1}\right) = 40 \\ \frac{2^2+3^2+4^2+x_1^2+x_2^2}{5} &= \frac{2\theta^2}{(\alpha-1)(\alpha-2)} \\ 29+x_1^2+x_2^2 &= 5\left(\frac{2(373.71^2)}{(46.71)(45.71)}\right) = 654 \end{aligned}$$

We use the first equation to solve for x_2 in terms of x_1, and plug that into the second equation and solve.

$$\begin{aligned} x_2 &= 31 - x_1 \\ 29 + x_1^2 + (31-x_1)^2 &= 654 \\ 29 + 2x_1^2 - 62x_1 + 961 &= 654 \\ 2x_1^2 - 62x_1 + 336 &= 0 \\ x_1^2 - 31x_1 + 168 &= 0 \\ x_1 &= \frac{31 \pm \sqrt{31^2 - 4(168)}}{2} \\ &= \frac{31 \pm 17}{2} = 7 \text{ or } 24 \end{aligned}$$

Since x_2 is higher than x_1, $x_1 = \boxed{\mathbf{7}}$. **(C)**

16. [**Lesson 48**] The hypothetical mean is Λ. The expected hypothetical mean $\mu = \mathbf{E}[\Lambda] = 1$ (the mean of the uniform distribution). The process variance is Λ. The expected process variance v, or the expected value of Λ, is 1. The variance of the hypothetical mean is $\mathrm{Var}(\Lambda)$. For a uniform distribution on $(0,\theta)$, the variance is $\frac{\theta^2}{12}$, so the variance is $a = \frac{1}{3}$. The Bühlmann k is therefore $\frac{1}{1/3} = 3$. $Z = \frac{1}{1+3} = 0.25$. The credibility premium is $\frac{1}{4}(5) + \frac{3}{4}(1) = \boxed{\mathbf{2}}$. **(C)**

17. [**Lesson 57**] This is a binomial distribution with $m = 100$, $q = 0.01$.

$$p_0 = 0.99^{100} = 0.366$$
$$p_1 = 100\left(0.99^{99}\right)(0.01) = 0.370$$

Together these add up to more than 0.68, the highest uniform number. 0.12 and 0.35 go to 0 and 0.68 goes to 1. $0+0+1=\boxed{1}$. **(B)**

18. [**Lesson 22 and section 23.2**] First we calculate $\hat{S}(26)$.

y_i	r_i	s_i	$S_{10}(y_i)$
5	10	1	0.9
8	8	1	0.7875
10	7	1	0.675
11	6	1	0.5625
17	5	1	0.45
20	4	1	0.3375
26	2	1	0.16875

So $\hat{S}(26) = 0.16875$. To extrapolate, we exponentiate $\hat{S}(26)$ to the $\frac{30}{26}$ power, as discussed in example 22E on page 351:

$$\hat{S}(30) = 0.16875^{30/26} = 0.12834.$$

$\Pr(20 \le T \le 30) = S(20^-) - S(30)$, since the lower endpoint is included. But $S(20^-) = S(17) = 0.45$. So the answer is $0.45 - 0.12834 = \boxed{0.3217}$. **(D)**

19. [**Section 4.1 and Lesson 11**] The density of the uniform distribution is the reciprocal of the range (1/2), or 2. We integrate p_0 for the binomial, or $(1-q)^2$, over the uniform distribution.

$$\begin{aligned}\Pr(N=0) &= \int_{0.25}^{0.75} (2)(1-q)^2\,dq \\ &= -2\left(\frac{(1-q)^3}{3}\right)\Bigg|_{0.25}^{0.75} \\ &= \left(\frac{2}{3}\right)(0.75^3 - 0.25^3) = \boxed{0.270833} \qquad \textbf{(D)}\end{aligned}$$

20. [**Lesson 54**] We estimate μ, v, and a:

$$\hat{\mu} = \hat{v} = \bar{x} = \frac{22(1)+6(2)+2(3)}{100} = \frac{40}{100} = 0.4$$

We will calculate s^2, the unbiased sample variance, by calculating the second moment, subtracting the square of the sample mean (which gets us the empirical variance) and then multiplying by $\frac{n}{n-1}$ to turn it into the sample variance.

$$\begin{aligned}\hat{a} + \hat{v} = s^2 &= \frac{100}{99}\left(\frac{22(1^2)+6(2^2)+2(3^2)}{100} - \bar{x}^2\right) \\ &= \frac{100}{99}(0.64 - 0.4^2) = 0.4848 \\ \hat{a} &= 0.4848 - 0.4 = 0.0848\end{aligned}$$

$$Z = \frac{a}{a+v} = \frac{0.0848}{0.4848}$$
$$P_C = 0.4 + \frac{0.0848}{0.4848}(1.6) = \boxed{\mathbf{0.68}} \qquad \textbf{(E)}$$

21. [**Lesson 56**] $F_X(500)$ is approximately 0.4 based on the fifty runs. The standard deviation of $\hat{F}_X(500)$ is therefore $\sqrt{(0.4)(0.6)/n}$. We want the half-width of the confidence interval equal to 5% of 0.4, or

$$1.96\sqrt{(0.4)(0.6)/n} = 0.05(0.4)$$
$$\frac{0.9602}{0.02} = \sqrt{n}$$
$$n = 2304.96$$

$\boxed{\mathbf{2305}}$ runs are needed. **(E)**

22. [**Lessons 12 and 14**] Let N be claim counts, X claim size, S aggregate claims. N is a gamma mixture of a Poisson, or a negative binomial. The gamma has parameters α and θ (not the same θ as the Weibull) such that

$$\alpha\theta = 0.5$$
$$\alpha\theta^2 = 1$$

implying $\beta = \theta = 2$, $r = \alpha = 0.25$, so

$$\mathbf{E}[N] = r\beta = 0.5$$
$$\mathrm{Var}(N) = r\beta(1+\beta) = 0.25(2)(3) = 1.5$$

The Weibull[1] has mean

$$\mathbf{E}[X] = \theta\Gamma(1+2) = (5)(2!) = (5)(2) = 10$$

and second moment

$$\mathbf{E}\left[X^2\right] = \theta^2\Gamma(1+2^2) = (5^2)(4!) = (25)(24) = 600$$

and therefore $\mathrm{Var}(X) = 600 - 10^2 = 500$. By the compound variance formula

$$\mathrm{Var}(S) = (0.5)(500) + (1.5)(10^2) = \boxed{\mathbf{400}} \qquad \textbf{(D)}$$

23. [**Lesson 34**] We set up a table for the empirical and fitted functions. Note that we do not know the empirical function at 10,000 or higher due to the policy limit.

x	$F_n(x^-)$	$F_n(x)$	$F(x)$	Largest difference
1000	0	0.1	$1-e^{-1/6} = 0.1535$	0.1535
2000	0.1	0.4	$1-e^{-1/3} = 0.2835$	0.1835
4000	0.4	0.5	$1-e^{-2/3} = 0.4866$	0.0866
5000	0.5	0.7	$1-e^{-5/6} = 0.5654$	0.1346
10,000	0.7		$1-e^{-5/3} = 0.8111$	0.1111

Inspection indicates that the maximum difference occurs at 2000 and is $\boxed{\mathbf{0.1835}}$. **(D)**

[1] In general, $\Gamma(n) = (n-1)!$ for n an integer

24. [**Lesson 30**] See the discussion of transformations and the lognormal Example 30C on page 520, and the paragraph before the example. *The shortcut for lognormals must be adapted for this question since μ is given; it is incorrect to calculate the empirical mean and variance as if μ were unknown.* We will log each of the claim sizes and fit them to a normal distribution. You may happen to know that for a normal distribution, the MLE's of μ and σ are independent, so given μ, the MLE for σ will be the same as if μ were not given. Moreover, the MLE for σ for a normal distribution is the sample variance divided by n (rather than by $n-1$). If you know these two facts, you can calculate the MLE for σ on the spot. If not, it is not too hard to derive. The likelihood function (omitting the constant $\sqrt{2\pi}$) is (where we let $x_i =$ the *log* of the claim size)

$$\begin{aligned}
L(\sigma) &= \frac{1}{\sigma^5}\prod_{i=1}^{5} e^{-\frac{(x_i-7)^2}{2\sigma^2}} \\
l(\sigma) &= -5\ln\sigma - \sum_{i=1}^{5}\frac{(x_i-7)^2}{2\sigma^2} \\
\frac{\mathrm{d}l}{\mathrm{d}\sigma} &= -\frac{5}{\sigma} + \frac{\sum_{i=1}^{5}(x_i-7)^2}{\sigma^3} = 0 \\
\sigma^2 &= \frac{\sum_{i=1}^{5}(x_i-7)^2}{5}
\end{aligned}$$

We calculate

$$\sigma^2 = \frac{(\ln 400-7)^2 + 2(\ln 1100-7)^2 + (\ln 3000-7)^2 + (\ln 8000-7)^2}{5} = 1.1958$$

The mean of the lognormal is

$$\exp\left(\mu+\sigma^2/2\right) = \exp(7+1.1958/2) = \boxed{\mathbf{1994}} \qquad \textbf{(A)}$$

25. [**Lesson 25**] The kernel survival function for a uniform kernel is a straight line from 1 to 0, starting at the observation point minus the bandwidth and ending at the observation point plus the bandwidth. From the perspective of 74, we reverse orientation; the kernel survival for 74 *increases* as the observation increases. Therefore, the kernels are 0 at 70, $\frac{1}{8}$ at 72 (which is $\frac{1}{8}$ of the way from 71 to 79), $\frac{3}{8}$ at 74 (which is $\frac{3}{8}$ of the way from 71 to 79), and $\frac{1}{2}$ at 75 (which is $\frac{1}{2}$ of the way from 71 to 79). Each point has a weight of $\frac{1}{n} = \frac{1}{5}$. We therefore have:

$$\frac{1}{5}\left(\frac{1}{8}+\frac{3}{8}+(2)\left(\frac{1}{2}\right)\right) = \boxed{\mathbf{0.30}} \qquad \textbf{(B)}$$

26. [**Lesson 29**] The likelihood function is the product of

$$e^{-\theta}\frac{\theta^{n_i}}{n_i!}$$

for the number of claims n_i for the 4 individuals, times the product of

$$\frac{1}{1000\theta}e^{-x_i/1000\theta}$$

for each of the 10 claim sizes x_i. These get multiplied together to form the likelihood function. We have

$$\sum n_i = 4+1+2+3 = 10$$

and

$$\sum \frac{x_i}{1000\theta} = \frac{2000+4000+4000+7000+4000+2000+3000+1000+4000+5000}{1000\theta} = \frac{36}{\theta}$$

If we ignore the constants, the likelihood function is:

$$L(\theta) = e^{-4\theta}\theta^{10}\frac{1}{\theta^{10}}e^{-36/\theta}$$
$$l(\theta) = -4\theta - \frac{36}{\theta}$$
$$\frac{\mathrm{d}l}{\mathrm{d}\theta} = -4 + \frac{36}{\theta^2} = 0$$
$$\theta = 3$$

To complete the answer to the question, use equation (14.2), or better, since number of claims is Poisson, equation (14.4). Let S be aggregate losses. Using either formula, we obtain $\mathrm{Var}(S) = 3\bigl(2(3000^2)\bigr) = \boxed{\mathbf{54{,}000{,}000}}$ for the fitted distribution. **(E)**

27. [**Lessons 12 and 13**] The mixed number of claims for all risks is negative binomial with $r = 3$, $\beta = 0.1$. However, this must be adjusted for severity modification; only $F(10{,}000) = 1 - \left(\frac{20{,}000}{30{,}000}\right)^3 = \frac{19}{27}$ of claims are handled by your department, where F is the distribution function of a Pareto. The modification is to set $\beta = \frac{19}{27}(0.1)$. The variance is then $r\beta(1+\beta) = 3\left(\frac{1.9}{27}\right)\left(1 + \left(\frac{1.9}{27}\right)\right) = \boxed{\mathbf{0.2260}}$. **(C)**

28. [**Subsection 53.2**] We apply formulas (53.5) and (53.6).

$$\bar{x}_1 = \frac{1000+1200}{40+50} = 24\tfrac{4}{9}$$
$$\bar{x}_2 = \frac{500+600}{20+40} = 18\tfrac{1}{3}$$
$$\bar{x} = \frac{1000+1200+500+600}{40+50+20+40} = \frac{3300}{150} = 22$$
$$\hat{v} = \frac{40\left(\frac{1000}{40} - 24\frac{4}{9}\right)^2 + 50\left(\frac{1200}{50} - 24\frac{4}{9}\right)^2 + 20\left(\frac{500}{20} - 18\frac{1}{3}\right)^2 + 40\left(\frac{600}{40} - 18\frac{1}{3}\right)^2}{2}$$
$$= 677\tfrac{7}{9}$$
$$\hat{a} = \frac{1}{150 - \frac{1}{150}(90^2+60^2)}\left(90(24\tfrac{4}{9} - 22)^2 + 60(18\tfrac{1}{3} - 22)^2 - (677\tfrac{7}{9})(1)\right)$$
$$= \frac{666\frac{2}{3}}{72} = 9.2593$$
$$\hat{k} = \frac{\hat{v}}{\hat{a}} = \frac{677\frac{7}{9}}{9.2593} = 73.2$$
$$\hat{Z}_1 = \frac{90}{90+\hat{k}} = \frac{90}{90+73.2} = \boxed{\mathbf{0.5515}} \qquad \textbf{(E)}$$

29. [**Lesson 49**] For aggregate losses, the mean given θ is $\frac{\theta}{3}$ and the variance given θ is $\frac{2\theta^2}{6} - \left(\frac{\theta}{3}\right)^2 = \frac{2\theta^2}{9}$. Then

$$v = \frac{2}{9}\mathbf{E}\left[\theta^2\right] = \frac{2}{9}(\mu^2+\sigma^2) = \frac{2}{9}(3^2+1^2) = \frac{20}{9}$$
$$a = \frac{1}{9}\mathrm{Var}(\theta) = \frac{1}{9}\sigma^2 = \frac{1}{9}$$

$$Z = \frac{a}{a+v} = \boxed{\frac{1}{21}} \qquad \textbf{(A)}$$

30. [**Lesson 9**] Let X be loss size. Since $F(0)=0$, $\mathbf{E}[X]=e(0)=3000$. Then

$$\begin{aligned}
\mathbf{E}[X] &= \mathbf{E}[X\wedge d] + e(d)\big(1-F(d)\big)\\
3000 &= \mathbf{E}[X\wedge 500] + e(500)\big(1-F(500)\big)\\
&= \mathbf{E}[X\wedge 500] + (2800)(0.9)\\
\mathbf{E}[X\wedge 500] &= 480\\
3000 &= \mathbf{E}[X\wedge 10{,}000] + e(10{,}000)\big(1-F(10{,}000)\big)\\
&= \mathbf{E}[X\wedge 10{,}000] + (2600)(0.2)\\
\mathbf{E}[X\wedge 10{,}000] &= 2480\\
\mathbf{E}[X\wedge 10{,}000] - \mathbf{E}[X\wedge 500] &= 2480-480 = \boxed{\mathbf{2000}} \qquad \textbf{(D)}
\end{aligned}$$

31. [**Lesson 29**] The claims are truncated, not censored, at 10,000. The probability of seeing any claim is $F(10{,}000)$. Any likelihood developed before considering this condition must be divided by this condition.

The likelihood of each of the 100 claims less than 1000, if not for the condition, is $F(1000)$. The conditional likelihood, conditional on the claim being below 10,000, is $\frac{F(1000)}{F(10{,}000)}$.

The likelihood of each of the 75 claims between 1000 and 5000, if not for the condition, is $F(5000)-F(1000)$. The conditional likelihood is $\frac{F(5000)-F(1000)}{F(10{,}000)}$.

The likelihood of each of the 25 claims between 5000 and 10,000, if not for the condition, is $F(10{,}000)-F(5000)$. The conditional likelihood is $\frac{F(10{,}000)-F(5000)}{F(10{,}000)}$.

Multiplying all these 200 likelihoods together we get answer **(B)**.

32. [**Lesson 19**] The sample mean is an unbiased estimator of the population mean, and if the population variance is finite (as it is if it has an exponential distribution), the sample mean is a consistent estimator of the population mean. (C) and (E) are therefore true. For a sample of size 2, the sample median is the sample mean, so (B) is true. (D) is proved in *Loss Models*. That leaves (A). **(A)** is false, because (B) is true and the median of an exponential is not the mean. In fact, it is the mean times $\ln 2$. So the sample median, which is an unbiased estimator of the mean, and therefore has an expected value of θ, does not have expected value $\theta\ln 2$, the value of the median.

33. [**Lesson 28**] For a mixture F is the weighted average of the F's of the individual distribution. The median of the mixture F is then the number m such that

$$wF_1(x) + (1-w)F_2(x) = 0.5$$

where w is the weight. Here, it is more convenient to use survival functions. (The median is the number m such that $S(m)=0.5$) $m=5$. We have:

$$\begin{aligned}
we^{-5/3} + (1-w)e^{-5/x} &= 0.5\\
w(e^{-5/3}-e^{-5/x}) &= 0.5 - e^{-5/x}\\
w &= \frac{0.5-e^{-5/x}}{e^{-5/3}-e^{-5/x}}
\end{aligned}$$

In order for this procedure to work, w must be between 0 and 1. Note that since $x > 3$, $-\frac{5}{3} < -\frac{5}{x}$, so the denominator is negative. For $w > 0$, we need

$$\begin{aligned}
0.5 - e^{-5/x} &< 0 \\
e^{-5/x} &> 0.5 \\
-5/x &> \ln 0.5 \\
5/x &< \ln 2 \\
x &> \frac{5}{\ln 2} = 7.2135
\end{aligned}$$

For $w < 1$, we need

$$\begin{aligned}
e^{-5/x} - 0.5 &< e^{-5/x} - e^{-5/3} \\
0.5 &> e^{-5/3} = 0.1889
\end{aligned}$$

and this is always true. So percentile matching works when $\boxed{x > 7.2135}$. **(E)**

34. [**Section 3.2**] We recognize X as inverse gamma with $\alpha = 3$, $\theta = 4$. Then $\mathbf{E}[X] = \frac{4}{3-1} = 2$ and $\mathbf{E}\left[X^2\right] = \frac{4^2}{(2)(1)} = 8$, so $\mathrm{Var}(X) = 8 - 2^2 = 4$, and $\bar{X}$ has mean 2 and variance 0.04. The normal approximation gives

$$\Pr(\bar{X} < 2.5) = \Phi\left(\frac{2.5-2}{\sqrt{0.04}}\right) = \Phi(2.5) = \boxed{0.9938} \qquad \textbf{(E)}$$

35. [**Lesson 8**] The 95th percentile of a normal distribution with parameters $\mu = 3$, $\sigma = 0.5$ is $3 + 1.645(0.5) = 3.8225$. Exponentiating, the 95th percentile of a lognormal distribution is $e^{3.8225} = \boxed{45.718}$. **(C)**