ASM Study Manual for Exam P, Three Practice Exams, Set 2
By Dr. Krzysztof M. Ostaszewski, FSA, CERA, CFA, MAAA (krzysio@krzysio.net)
Errata

Posted May 10, 2010
In Problem 15 in Practice Examination 25,
95% should be replaced by at least 96%.

Posted January 29, 2010
In Problem 10 in Practice Examination 26, answer choices should be:
A. 1184 B. 1218 C. 1380 D. 1465 E. 1493
and the last two sentences of the solution should read:
Finally, the standard deviation of X is $\sqrt{1.4019} \approx 1.1840$, and the standard deviation of $1000X$ is approximately 1184.
Answer A.

Posted December 28, 2009
The answer choices and the solution of Problem 16 in Practice Examination 25 should be:
A. 0.2500 B. 0.3333 C. 0.3442 D. 0.4264 E. 0.5000

Solution.
The total number of balls is 18. Total number of ways to pick 6 of them is $\binom{18}{6} = 18564$. We will calculate the probability of having all colors represented by finding the probabilities of each color not being represented and them subtracting their sum from 1. The number of ways to not pick the first color is $\binom{3}{0} \cdot \binom{15}{6}$, the number of ways to not pick the second color is $\binom{4}{0} \cdot \binom{14}{6}$, the number of ways not to pick the third color is $\binom{5}{0} \cdot \binom{13}{6}$, and finally, the number of ways to not pick the fourth color is $\binom{6}{0} \cdot \binom{12}{6}$. Thus, the probability sought is:

$$1-\frac{\binom{3}{0}\cdot\binom{15}{6}+\binom{4}{0}\cdot\binom{14}{6}+\binom{5}{0}\cdot\binom{13}{6}+\binom{6}{0}\cdot\binom{12}{6}}{\binom{18}{6}}=$$

$$=1-\frac{\binom{15}{6}+\binom{14}{6}+\binom{13}{6}+\binom{12}{6}}{18564}=1-\frac{5005+3003+1716+924}{18564}\approx 0.4264.$$

Answer D.

Posted December 27, 2009

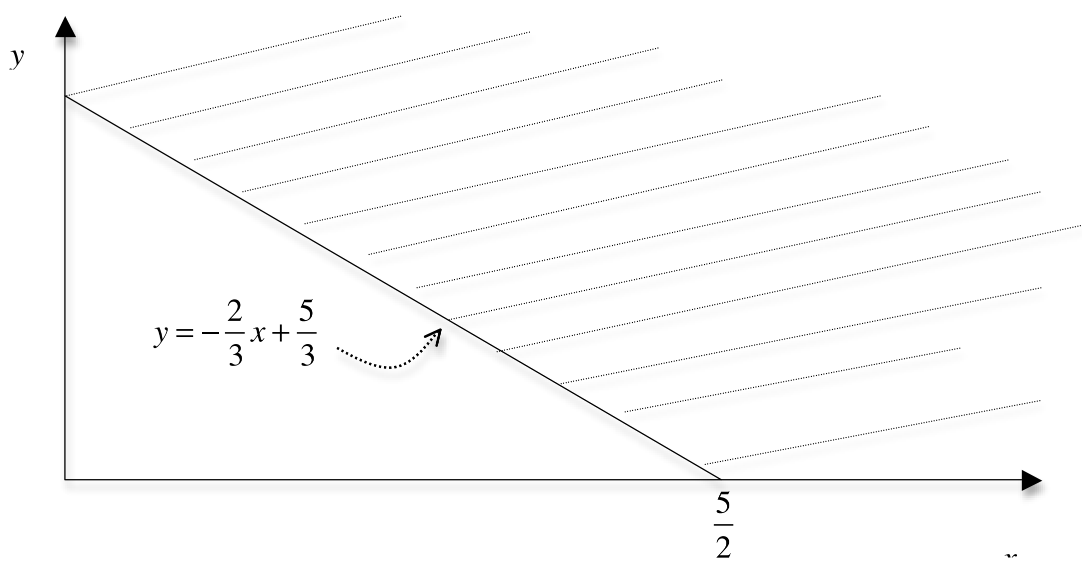
In Problem 1, Practice Examination 25, answer choices should be:

A. 0.0476 B. 0.0526 C. 0.0935 D. 0.1075 E. 0.1233

The solution should be replaced by the following:

Solution.

The joint distribution of X and Y is $f_{X,Y}(x,y)=2e^{-2x}\cdot 2e^{-2y}$ for $x>0$, $y>0$, and zero otherwise. We have $\Pr(2X+3Y>5)=\Pr\left(Y>-\frac{2}{3}X+\frac{5}{3}\right)$ and the region where $Y>-\frac{2}{3}X+\frac{5}{3}$ is shown in the figure below:



Therefore,

$$\Pr(2X+3Y>5)=\Pr\left(Y>-\frac{2}{3}X+\frac{5}{3}\right)=$$

$$=\int_0^{\frac{5}{2}}\left(\int_{-\frac{2}{3}x+\frac{5}{3}}^{+\infty} f_{X,Y}(x,y)\,dy\right)dx+\int_{\frac{5}{2}}^{+\infty}\left(\int_0^{+\infty} f_{X,Y}(x,y)\,dy\right)dx=$$

$$=\int_0^{\frac{5}{2}}\left(\int_{-\frac{2}{3}x+\frac{5}{3}}^{+\infty}\left(2e^{-2x}\cdot 2e^{-2y}\right)dy\right)dx+\int_{\frac{5}{2}}^{+\infty}\left(\int_0^{+\infty}\left(2e^{-2x}\cdot 2e^{-2y}\right)dy\right)dx.$$

We can calculate the two integrals separately. First,

$$\int_0^{\frac{5}{2}}\left(\int_{-\frac{2}{3}x+\frac{5}{3}}^{+\infty}\left(2e^{-2x}\cdot 2e^{-2y}\right)dy\right)dx=\int_0^{\frac{5}{2}}\left(\left(-2e^{-2x}\right)\int_{-\frac{2}{3}x+\frac{5}{3}}^{+\infty}\left(-2e^{-2y}\right)dy\right)dx=$$

$$=\int_0^{\frac{5}{2}}\left(\left(-2e^{-2x}\right)\left(e^{-2y}\Big|_{y=-\frac{2}{3}x+\frac{5}{3}}^{y\to\infty}\right)\right)dx=\int_0^{\frac{5}{2}}\left(\left(-2e^{-2x}\right)\left(-e^{\frac{4}{3}x-\frac{10}{3}}\right)\right)dx=$$

$$=2e^{-\frac{10}{3}}\int_0^{\frac{5}{2}}e^{-\frac{2}{3}x}\,dx=2e^{-\frac{10}{3}}\cdot\left(-\frac{3}{2}e^{-\frac{2}{3}x}\Big|_{x=0}^{x=\frac{5}{2}}\right)=3e^{-\frac{10}{3}}\cdot\left(1-e^{-\frac{5}{3}}\right)=3\left(e^{-\frac{10}{3}}-e^{-5}\right).$$

The second integral equals

$$\int_{\frac{5}{2}}^{+\infty}\left(\int_0^{+\infty}\left(2e^{-2x}\cdot 2e^{-2y}\right)dy\right)dx=\int_{\frac{5}{2}}^{+\infty}2e^{-2x}\cdot\underbrace{\left(\int_0^{+\infty}2e^{-2y}\,dy\right)}_{\substack{\text{Integral over all possible}\\\text{values of the PDF of}\\\text{exponential with hazard}\\\text{rate of 2, it equals 1}}}dx=\underbrace{\int_{\frac{5}{2}}^{+\infty}2e^{-2x}\,dx}_{\substack{\text{Probability that exponential}\\\text{with hazard rate 2 exceeds}\\\text{the value of }\frac{5}{2}}}=e^{-2\cdot\frac{5}{2}}=e^{-5}.$$

The total of the two equals

$$3\left(e^{-\frac{10}{3}}-e^{-5}\right)+e^{-5}=3e^{-\frac{10}{3}}-2e^{-5}\approx 0.0935.$$

Answer C.

Posted December 27, 2009

In Practice Examination 25, Problem 2, answer choices should be:

A. $\frac{d}{r}$ B. $\frac{d^2}{r^2}$ C. $1-\frac{6d^2}{r^2}$ D. $1-\frac{6d^2}{r^2}+\frac{8d^3}{r^3}$ E. $1-\frac{6d^2}{r^2}+\frac{8d^3}{r^3}-\frac{3}{r^4}$

The solution should be replaced by:

Solution.

Let us write X for the ground-up loss, which is given as distributed uniformly on $(0,r)$.

Therefore, we know that $\operatorname{Var}(X)=\frac{r^2}{12}$. Let us also write Y for the amount paid by the policy
with the deductible of d. We know that

$$E(Y)=\int_d^r s_X(x)\,dx=\int_d^r \frac{r-x}{r}\,dx=\int_d^r\left(1-\frac{x}{r}\right)dx=\left(x-\frac{x^2}{2r}\right)\Bigg|_{x=d}^{x=r}=r-\frac{r^2}{2r}-d+\frac{d^2}{2r}=$$

$$=(r-d)-\frac{1}{2r}\left(r^2-d^2\right)=(r-d)-\frac{(r-d)(r+d)}{2r}=(r-d)\left(1-\frac{r+d}{2r}\right)=\frac{(r-d)^2}{2r}.$$

Also,

$$E\left(Y^2\right)=\int_d^r (x-d)^2\cdot\frac{1}{r}\,dx=\frac{1}{r}\cdot\frac{1}{3}(x-d)^3\Bigg|_{x=d}^{x=r}=\frac{(r-d)^3}{3r}.$$

This gives

$$\operatorname{Var}(Y)=E\left(Y^2\right)-\left(E(Y)\right)^2=\frac{(r-d)^3}{3r}-\left(\frac{(r-d)^2}{2r}\right)^2=\frac{(r-d)^3}{3r}-\frac{(r-d)^4}{4r^2}=$$

$$=\frac{4r(r-d)^3-3(r-d)^4}{12r^2}=\frac{r^4-6r^2d^2+8rd^3-3d^4}{12r^2}.$$

The ratio of the variance of the insurance payment to the variance of the loss is

$$\frac{\operatorname{Var}(Y)}{\operatorname{Var}(X)}=\frac{\dfrac{r^4-6r^2d^2+8rd^3-3d^4}{12r^2}}{\dfrac{r^2}{12}}=\frac{r^4-6r^2d^2+8rd^3-3d^4}{r^4}=1-\frac{6d^2}{r^2}+\frac{8d^3}{r^3}-\frac{3}{r^4}.$$

Answer E.

Posted December 27, 2009
In the solution of Problem 14, Practice Examination 25, the formula:

$$F_{Y_{(1)},Y_{(4)}}(s,t) = \Pr\left(Y_{(1)} \le s, Y_{(4)} \le t\right) =$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t, \text{ but not all are greater than } s\}\right) =$$
$$= t^4 - (t-s)^4 = |\text{below is another development of the same formula}|$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and one exactly is less than } s\}\right) +$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and two exactly are less than } s\}\right) +$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and three exactly are less than } s\}\right) +$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and four exactly are less than } s\}\right) =$$
$$= \binom{4}{1} \cdot (t-s)^3 \cdot s + \binom{4}{2} \cdot (t-s)^2 \cdot s^2 + \binom{4}{3} \cdot (t-s) \cdot s^3 + \binom{4}{3} \cdot s^4 =$$
$$= t^4 - (t-s)^4 .$$

has three equal signs that should be pluses, and it should instead be:

$$F_{Y_{(1)},Y_{(4)}}(s,t) = \Pr\left(Y_{(1)} \le s, Y_{(4)} \le t\right) =$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t, \text{ but not all are greater than } s\}\right) =$$
$$= t^4 - (t-s)^4 = |\text{below is another development of the same formula}|$$
$$= \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and one exactly is less than } s\}\right) +$$
$$+ \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and two exactly are less than } s\}\right) +$$
$$+ \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and three exactly are less than } s\}\right) +$$
$$+ \Pr\left(\{\text{All of } X_1, X_2, X_3, X_4 \text{ are less than or equal } t \text{ and four exactly are less than } s\}\right) =$$
$$= \binom{4}{1} \cdot (t-s)^3 \cdot s + \binom{4}{2} \cdot (t-s)^2 \cdot s^2 + \binom{4}{3} \cdot (t-s) \cdot s^3 + \binom{4}{3} \cdot s^4 =$$
$$= t^4 - (t-s)^4 .$$

Posted December 25, 2009

The solution of Problem 5 in Practice Examination 24 should be replaced by:

Since $Y = \max(0, -X)$, Y is nonnegative with probability 1. This implies that $F_Y(y) = 0$ for $y < 0$. For $y \ge 0$,

$$F_Y(y) = \Pr(Y \le y) = \Pr(\max(0, -X) \le y) =$$
$$= \Pr\left(\{\max(0, -X) \le y\} \cap \{X < 0\}\right) + \Pr\left(\{\max(0, -X) \le y\} \cap \{X \ge 0\}\right) =$$
$$= \Pr\left(\{-X \le y\} \cap \{X < 0\}\right) + \Pr\left(\{0 \le y\} \cap \{X \ge 0\}\right) =$$
$$= \Pr(-y \le X < 0) + \Pr(X \ge 0) = \Pr(-y \le X) = \Pr(-y < X) = 1 - F_X(-y).$$

Answer E.

Posted December 22, 2009
In Practice Examination 24, Problem 3, the formula in the solution should be:

$$\#(D_m) = \#(P_m) - \sum_{i=1}^{m} \#(A_i) + \bigcup_{1 \le i_1 < i_2 \le m} \#\left(A_{i_1} \cap A_{i_2}\right) - \ldots + (-1)^m \cdot \#(A_1 \cap A_2 \cap \ldots \cap A_m) =$$

$$= m! - \binom{m}{1} \cdot (m-1)! + \binom{m}{2} \cdot (m-2)! - \ldots + (-1)^m \cdot \binom{m}{m} \cdot 0! =$$

$$= m! \cdot \left(1 - \frac{1}{1!} + \frac{1}{2!} - \ldots + (-1)^m \cdot \frac{1}{m!}\right).$$

instead of

$$\#(D_m) = \#(P_m) - \sum_{i=1}^{m} \#(A_i) + \bigcup_{1 \le i_1 < i_2 \le m} \#\left(A_{i_1} \cap A_{i_2}\right) - \ldots + (-1)^m \cdot \#(A_1 \cap A_2 \cap \ldots \cap A_m) =$$

$$= m! - \binom{m}{1} \cdot (m-1)! + \binom{m}{2} \cdot (m-2)! - \ldots + (-1)^m \cdot \binom{m}{m} \cdot 1! =$$

$$= m! \cdot \left(1 - \frac{1}{1!} + \frac{1}{2!} - \ldots + (-1)^m \cdot \frac{1}{m!}\right).$$

although, of course, this typo has no bearing on the calculation.

Posted December 17, 2009
In Practice Examination 25, Problem 27, the answer choices should be:

A. 2.25 B. 2.91 C. 3.85 D. 9.37 E. 12.50

Also, at the end of the solution, the expected value should be calculated as:

$$E(X) = 1 \cdot \frac{2}{15} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{2}{15} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{2}{15} + 6 \cdot \frac{1}{5} = \frac{18}{15} + \frac{12}{5} = \frac{54}{15},$$

and the variance should be calculated as:

$$\mathrm{Var}(X) = E\left(X^2\right) - \left(E(X)\right)^2 = \frac{238}{15} - \left(\frac{54}{15}\right)^2 = \frac{3570 - 2916}{225} = \frac{654}{225} \approx 2.9067.$$